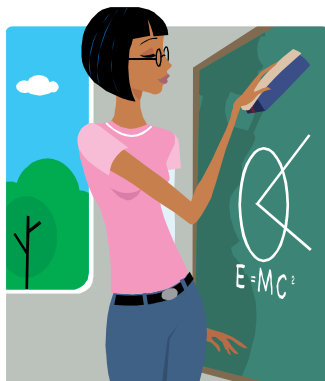


Chapter 9



Effective Practices in Mathematics: Specialty Supplies

Primary Author

Joan Córdova, Orange Coast College (faculty)

With special thanks to contributors from:

Donna Ames, MDTP

Stan Benkosky, West Valley College (faculty)

Wade Ellis, West Valley College (emeritus faculty)

Larry Green, Lake Tahoe Community College (faculty)

Barbara Illowsky, DeAnza College (faculty)

Alfred Manaster, Director of MDTP, UCSD

Diane Mathios, DeAnza College (faculty)

Ken Meehan, Fullerton College (researcher)

Bob Pacheco, Barstow College (faculty)

Terrie Teegarden, San Diego Mesa College (faculty)

Chaffey College Mathematics Department

Coastline Community College Mathematics Department

Golden West College Math Department (faculty)

Lake Tahoe Community College Mathematics Department

College of San Mateo Mathematics Department

Solano Community College Mathematics Department

San Diego Mesa College Math Department and

And reading and math expertise from

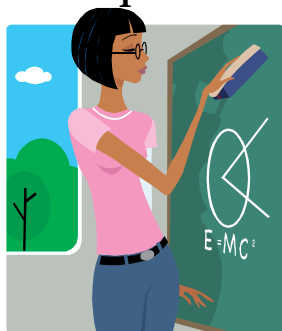
Lynn Hargrove, Sierra College Faculty

Dianne McKay, Mission College Faculty

Sara Pries, Sierra College Faculty

Lisa Rochford, Sierra College Faculty

Chapter 9



Effective Practices in Mathematics: Specialty Supplies

Introduction to New Perspectives in Teaching Mathematics

Welcome to the Mathematic Builders Emporium! This chapter contains a selection of construction materials that mathematics faculty from across the state have found to be effective in helping students with basic skills needs build their house of academic dreams. The supplies range from planking and sheetrock for active learning to the nuts and bolts of classroom assessment techniques. Research has shown that when instructing adult learners, teachers must actively involve participants in the learning process and serve as facilitators for them. Cognitively Guided Instruction, Authentic Assessment, Classroom Assessment Techniques (CATS) or a variety of active learning strategies are all effective means of delivering curriculum this way. In addition, an added bonus with these techniques is that they enable the instructor to get immediate feedback as to whether the content is being understood by the student or, to use our construction metaphor, whether the student is able to use a hammer to actually build. A variety of building methods are presented here because mathematics faculty have reached no consensus about one best approach; try several and see which works best for you and your students. If we've neglected to profile a strategy that you've found to be effective, submit it to: <http://bsi.cccco.edu/>. This will enable us to share the wealth and supplies with faculty across the state!

At the Emporium, we've found that the most successful teachers combine effective practices with self analysis or assessment to determine what pedagogical techniques work best. It's best to analyze how your materials work and the soundness of what they construct as you're building. The Final Report of the National Mathematics Advisory Panel concluded that,

Teachers who consistently produce significant gains in students' mathematics achievement can be identified using value-added analyses (analyses that examine individual students' achievement gains as a function of the teacher). The impact on students' mathematics learning is compounded if students have a series of these more effective teachers. (U.S. Department of Education, 2008, p. xx)

An analysis of classroom pedagogy, use of student learning outcomes assessments and active hands-on learning by Graves found that teaching mathematics in context is essential to student success (Graves, 1998, p. 2). Grave's analysis of hands-on, objective-based, contextual mathematics concluded that 70% of the students in the developmental mathematics cohort reported that this type of teaching helped them understand mathematics concepts better than any previous courses (Graves, 1998, p. 22). In addition, Grave, using pre- and post-testing, reported a 26.2% increase in

fundamental mathematics course performance and a 90% increase in algebra course performance (Graves, 1998, p. 23). Because mathematics is often referred to as a gatekeeper course, potentially creating a barrier to student post secondary success, examining effective practices in mathematics is an essential element to overall basic skills success. If you are new to understanding and documenting desired student learning outcomes and ongoing assessment to improve pedagogical strategies you may want to skip to Chapter 15 of this handbook on Course Assessment Basics: Evaluating Your Construction.

You'll notice that there are several aisles in the Emporium devoted to contextual mathematics or mathematics in context. These supplies bridge the gap between abstract mathematical concepts and real-world applications. Helping students to succeed in any course relies on the students' ability to make connections with their world perspective and needs (Bransford, 1999, pp. 12-13; Graves, 1998, p. 20). Connections are a key feature—connections among topics, connections to other disciplines, and connections between mathematics and meaningful problems in the real world. Contextual mathematics emphasizes the dynamic, active nature of mathematics and the way mathematics enables students to make sense of their world. Students are encouraged to explore mathematical relationships, to develop and explain their own reasoning and strategies for solving problems and to use problem-solving tools appropriately. In *Developmental Mathematics: A Curriculum Evaluation* research indicates that making those connections will increase student success.

The overall results of the evaluation indicated an increase in students' academic performance in mathematics. Responses also indicated an improvement in both students' and instructors' attitudes about learning and teaching mathematics using real-life applications. This was demonstrated by their increased confidence and decreased frustration. (Graves, 1998, p.2)

Another important aspect of mathematics success involves student's examining their own attitudes and misconceptions in mathematics. An extensive discussion about student misconceptions and metacognition (examining their own learning) is covered in Chapter 5 of this handbook. If your students are operating under misconceptions about mathematics or misconceptions about their ability to learn mathematics, they will not be able to learn the correct information until they have dealt with these misconceptions (National Council of Teachers of Mathematics, 2000, p. 73). Recently mathematics faculty at a regional Basic Skills Initiative meeting created a list of typical misconceptions in their students. Do any sound familiar?

**Some Typical Misconceptions of Mathematics Students in California
Community Colleges**

I'll never use this in real life!

I can create my own math, it is new math.

When I am confused in math; you should read – I really don't have to do this!

If I do a Career Technical pathway, I don't need math.

I can't act like math is interesting, it's not cool.

I don't need a sentence, that's English class.

I did the problem in my head. I don't need to show my work.

$\frac{1}{4} + \frac{2}{3} = \frac{3}{7}$ or $\frac{3}{x+3} = 0/X = X$ or -2^2 vs $(-2)^2$ or $X - 1 = -x$



Let's start with a Quiz

What do you already know about effective building supplies for students with basic skills needs in mathematics? Mark the best answer to the questions below.

1. Students with basic skills needs are more likely to sign up for which of the following courses?
 - a. Pre-collegiate or basic skills English courses
 - b. Pre-collegiate or basic skills mathematics courses
 - c. Pre-collegiate or basic skills reading courses
 - d. Pre-collegiate or basic skills study skills courses
 - e. Pre-collegiate or basic skills science courses
2. The California community college success rate (meaning an A,B,C, or pass) for students in pre-collegiate or basic skills mathematics courses in Spring 2007 was approximately
 - a. 35 - 39 %
 - b. 40 - 44%
 - c. 45 - 49%
 - d. 50 - 54%
 - e. 55 - 60%
3. Select the equation that best represents the relationship of the success rates in the following basic skills courses?
 - a. Success in basic skills mathematics courses $>$ success in basic skills English courses $>$ success in basic skills ESL courses
 - b. Success in basic skills English courses $>$ success in basic skills mathematics courses $>$ success in basic skills ESL courses
 - c. Success in basic skills ESL $>$ success in basic skills English courses $>$ success in basic skills mathematics courses
 - d. Success in basic skills mathematics courses = success in basic skills English courses = success in basic skills ESL courses
 - e. Success in basic skills ESL courses $>$ success in basic skills mathematics courses = success in basic skills English courses
4. Which of the following strategies have been shown by research to contribute to success in basic skills mathematics courses?
 - a. contextualizing the mathematics to real world applications
 - b. smaller class size associated with feedback and interaction
 - c. active learning strategies where students must facilitate their own learning
 - d. high expectations for student responsibility
 - e. all of the above
5. The ICAS (Intersegmental Committee of Academic Senates) mathematics competencies
 - a. represent standardized education like No Child Left Behind
 - b. are the skills that students must routinely exercise without hesitation in order to be prepared for college work as agreed upon by the community colleges, California State Universities (CSU) and University of California (UC) faculty
 - c. are the learning outcomes for high school level mathematics
 - d. include competencies for all grade level mathematics courses
 - e. all of the above

6. What percent of students who assess into basic skills mathematics three levels below college ever make it to college level mathematics?
 - a. Less than 10%
 - b. Less than 25%
 - c. Less than 40%
 - d. Less than 50%
 - e. Greater than 50%

7. Which of the following have occurred where student learning outcomes have been created for mathematics classes?
 - a. Faculty, both full-time and part-time, are focused on the same outcomes but are free to use their own techniques to help student gain mathematics skills
 - b. Courses have been aligned within a mathematics sequence
 - c. Mathematics pre-requisites can be more easily validated for non-mathematics courses requiring pre-requisite mathematics knowledge
 - d. Program review and program level assessments have been simplified
 - e. All of the above

8. Research has shown that students' ability in mathematics is often
 - a. independent of reading and study skills
 - b. directly related to whether the student is a male or female
 - c. is impossible for dyslexic students
 - d. very dependent upon students' literacy or reading skills
 - e. all of the above

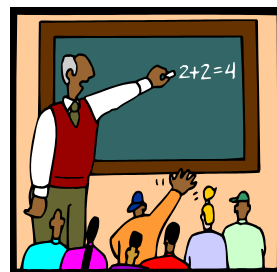
9. Which of the following are true of rubrics?
 - a. Rubrics are only used in writing and oral student work
 - b. Rubrics are used only by K-12 teachers
 - c. Rubrics are only used to grade student work
 - d. Creating a rubric defines the expectations and criteria for student work in any discipline
 - e. Rubrics define criteria and expectations for student work allowing the student to evaluate his or her own work and the faculty member to grade more uniformly

10. Student success in basic skills mathematics courses can be improved and has been exemplified in several California community college research programs.
 - a. True
 - b. False

Please see Appendix 1 for the answers to the quiz.

A Place for All Mathematics Students and Faculty to Start

One effective learning strategy for problem solving in mathematics is George Pólya's four-step problem-solving process. Pólya was a world-renowned mathematician made famous by his common sense approach to all problem-solving by constructing a framework of inquiry and experimentation (Pólya, 1957, pp. 5-6).



George Pólya's Four-Step Problem-Solving Process

- 1. Understanding the problem*
- 2. Developing a plan to solve the problem*
- 3. Carrying out the plan*
- 4. Looking back to be sure the answer solves the problem*

These steps apply not only to mathematics and other academic areas but also to life skills. They will be our guide through the chapter and Mathematics Emporium to discuss the difficulties that students with basic skills needs have with mathematics and developing ways to solve the problem. We begin with Pólya's step 1: understanding the problem.

1. Understanding the problem

Problem Summary: Postsecondary students are more likely to enroll in a remedial mathematics than in a remedial reading or writing class. The failure rate in these courses is alarming. Fewer than one-half are successful on their first attempt. **This overall success rate of 48% acts as a significant barrier to college success for students with basic skills needs** (CCCCO, 2008, MIS Datamart information).

The very low success rate in basic skills mathematics is a great barrier because as a gate keeper course, it can make or break a college career. In addition, employers identify deficient basic mathematics skills as common, particularly in students graduating from high school (The Conference Board, 2006, p. 13). The source of the problem is as varied as the number of students in a class. As you probably know from your own classroom and discussion with colleagues, students at the basic skills levels in mathematics have a variety of misconceptions, missed conceptions, and under-developed skills about your discipline.

If you're not sure about this, why not take a simple poll of the students in your classes and ask about their high school preparation? You will, perhaps, not be surprised to learn that often their

secondary education experience in mathematics was poor or non-existent, even in California where all students who graduate from high school must have three years of mathematics beyond pre-algebra. Though they took those classes, their learning experience may have consisted of sitting through context-free blackboard presentations and completing worksheets. These students are not accustomed to disciplined study of mathematics or any other subject, often they may not have experienced much success in any academic work. These students believe that they can only learn what the teacher tells them and have little experience engaging with interesting mathematical ideas, working from self-motivation or successful learning in an academic setting. They also miss the opportunity to apply what they know to their everyday world.

**California Community
College Mathematics
Success Rates:**

**48 % in Basic Skills
Mathematics Courses**

**53% in Non-basic Skills
Mathematics courses**

Statewide data shows that in California community colleges, the lowest success rate in basic skills courses is found in mathematics courses. This is also a common finding nation-wide. In addition, several studies have indicated that mathematics represent the roadblock for most students with basic skills needs (Graves, 1998, p.22). However, even with these low success rates, some of the most significant pedagogical successes have also been found through using innovative mathematics strategies.

To review effective improvement strategies for mathematics success, check out the MAPS process and Summer Math Jam discussed later in this chapter. See Appendix 2 for three articles on the Pathways through Algebra project with recommendations, successful strategies and data based on California community college students. Review Grave’s article cited in the references for the Mathematics Foundations Curricula.

The pedagogical strategies discussed in the Pathways Project and throughout this chapter have improved student success in mathematics. By success, we mean that students achieve a C or better in the course. Now consider the course success rate from your own classes, how often do students drop and what percentage routinely pass the course? This is important information for you to know and to work with.

“But,” you protest, “several variables contribute to the course success rate, and most of them are beyond my control. One of them,” you continue, “is that the students have poor study skills or any other number of challenges completely unrelated to math.” You are correct, in fact several factors contribute to student poor success in mathematics. They include but are not limited to:

**INCREASING
SUCCESS RATES**

**Some strategies have
shown increased
mathematics success
e.g.**

Mission College -MAPS

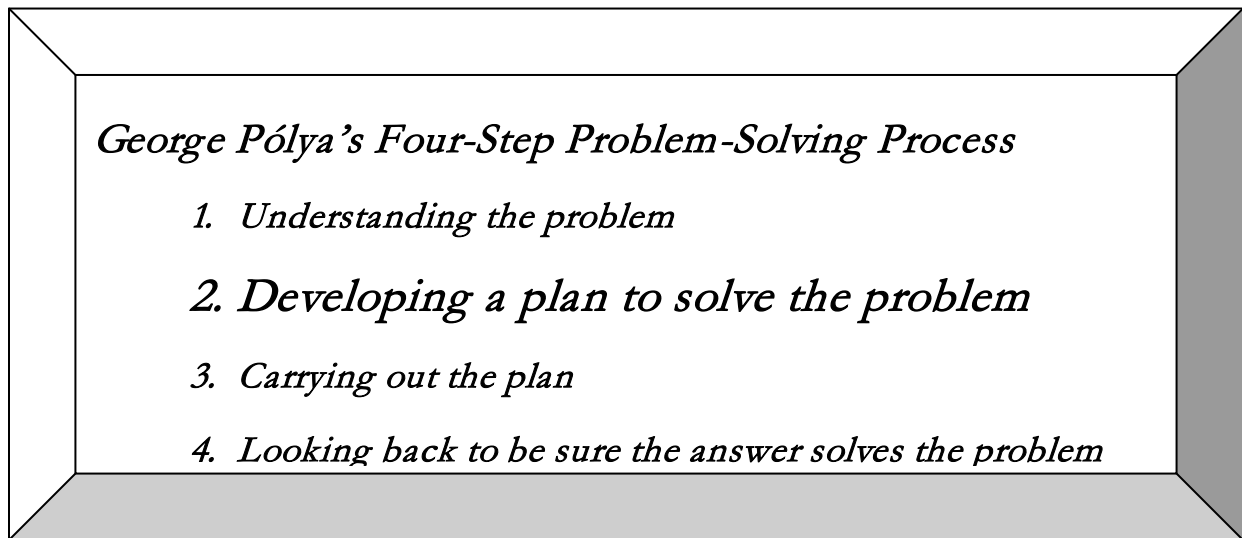
**Pathways through
Algebra**

**Mathematics
Foundations**

**Pasadena City College -
Summer MATH JAM**

1. Poor academic skills and learner identity
2. Lack of adequate counseling
3. Lack of adequate time devoted to learn due to low income requiring employment
4. Early childhood misconceptions
5. Attitudinal problems
6. Learning disabilities
7. Lack of career aspirations
8. Limited vision to see how mathematics connects to their life and future
9. Lack of maturity
10. Math anxiety

At this point, if you have skipped over Chapter 5 of this handbook, you might want to go back and examine some of the handy tools that faculty have created to address these student-based issues such as student responsibility and metacognition. Perhaps you can incorporate these tools into your courses. However, we also know from research that sometimes student success is related to curriculum alignment, active learning strategies, and contextualized curriculum. SO let's see if we can focus on some of the discipline-specific problem-solving strategies. Returning to Pólya's step 2: developing a plan to solve the problem, what do we know that can create better student learning and increased student success in mathematics courses?



2. Developing a plan to solve the problem

Plan Summary: Research shows class size plus active learning is an effective strategy to raise student success. The plan to accomplish this must be three fold:

- a) Implement active and contextualized learning strategies in classes with appropriate class sizes,
- b) Clearly communicate the college level competencies including the students' own responsibilities in mathematics classes, and
- c) Design a course around appropriate SLOs that align with subsequent courses.

A. Implement Active and Contextualized Learning Strategies

Class size + Active Learning = Success in Basic Skills Mathematics

The formula above has been supported by research and represents an important overarching strategy to solve the lack of success in mathematics and to increase and improve student learning (Center for Public Education, 2006). Class size is a very important factor in mathematics success. Of course success is not based on class size alone. A small class without active learning will show the same lack of success. However, with the appropriate class size, basic skills mathematics instructors who engage their students in active learning have been able to inspire and awaken students with basic skills needs to the joys of mathematics or, in the least, produce better mathematics success. These are instructors who create classrooms that are socially comfortable, context-based in relevant or interesting ideas, and actively engage students in thinking about and understanding mathematics. As an added bonus, these experiences can also provide the students with immediate feedback that both corrects their content knowledge and skills and helps them improve their learning processes. This, in turn, affects their studies in many areas.

In addition to active learning and assessment strategies, make clear expectations concerning student responsibilities. How do we make those expectations clear and communicate student learning outcomes? In Appendix 3 is a simple tool, a mathematics study skills inventory that you can provide to your students on the first day of class as you are taking roll. This documents expectations for mathematics students. Some faculty reported that they use this strategy as a discussion on the first week of class to norm the students' perception of what is expected and to allow students to reveal any misconceptions they have with regards to these components of learning. When students discover that these expectations lead to success and are accepted by their fellow students, it begins a metacognitive process as they evaluate their own responsibility and performance. We would like to add another simple visual tool to share with your students.

“That is all well and good,” you might be thinking, “but I already stress all of that. It’s in my syllabus, and I talk about it in class. As for the rest of that formula for success, my college or the district decides my class size; I don’t have control over it. And I’m already using active learning strategies, at least as often as I can while still trying to cover the material that’s required for each level.”

Great! We know that you are working hard, and that sometimes your colleagues who teach higher levels of mathematics may not appreciate all that you do. We hope that the specific pedagogical strategies listed later in this chapter will provide you with additional items to add to your toolbox. You may also want to peruse the two websites listed below from professional organizations that are looking at effective pedagogical techniques:

- Amatyc found at <http://www.amatyc.org/>
- Developmental Math website for Amatyc <http://www.devmath.amatyc.org/>

We asked a faculty member with over 30 years experience of teaching basic skills mathematics:

“If you could think of words to describe a successful basic math student, what would they be?”

He lifted his palm to us and said,

“The Big Five:

Come to class

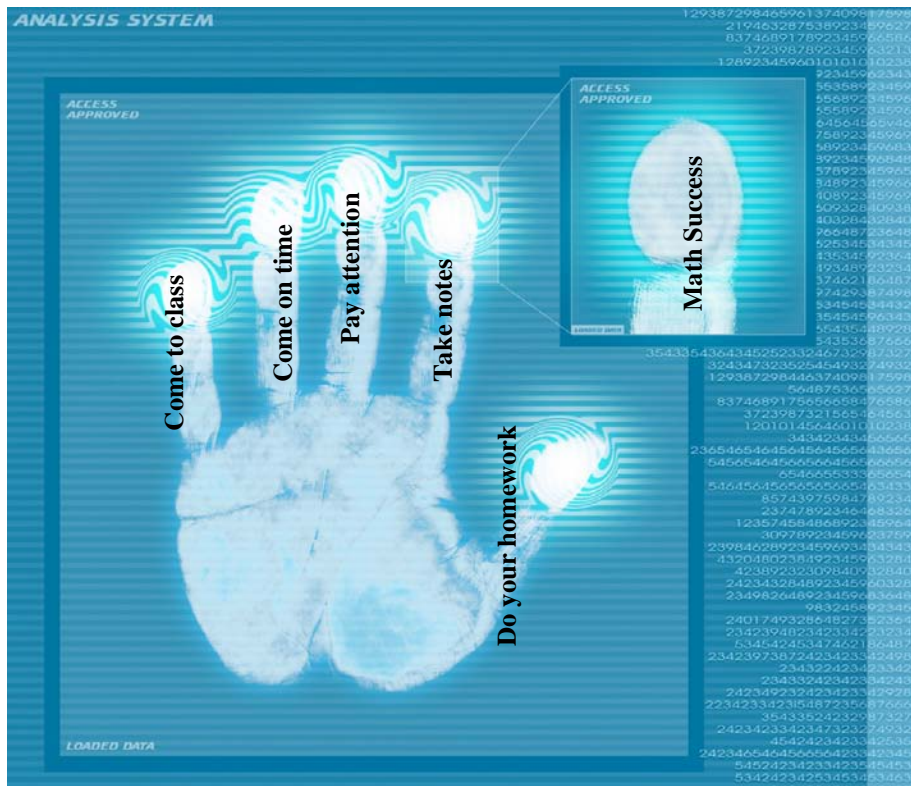
Come on time

Pay attention

Take notes,

Do your homework.”

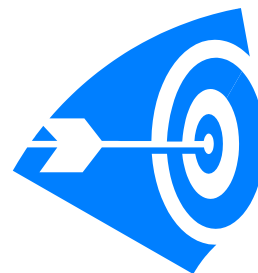
Original concept credited to Stan Benkosky Mathematics Faculty at West Valley Mission



B. Communicate College Level Mathematics Competencies

What is needed to solve the problem?

First a target is necessary so that we are all aiming at the same outcomes. The CSU, UC and CCC academic senates have adopted “college level” mathematics competencies called the ICAS (Intersegmental Committee of Academic Senates) Competencies. Mathematics classes should align to build towards these competencies. This is our target as instructors and the students’ target that identifies them as prepared for college level mathematics. Our classes and our curricula should cover these areas and regularly assess the student’s abilities to complete tasks associated with these competencies.



ICAS Statement of Mathematics Competencies (also in Appendix 4)

What follows is a collection of skills that students must routinely exercise without hesitation in order to be prepared for college work. These are intended as indicators -- students who have difficulty with many of these skills are significantly disadvantaged and are apt to require remediation in order to succeed in college courses. This list is not exhaustive of the basic skills. This is also not a list of skills that are sufficient to ensure success in college mathematical endeavors.

The absence of errors in student work is not the litmus test for mathematical preparation. Many capable students will make occasional errors in performing the skills listed below, but they should be

in the habit of checking their work and thus readily recognize these mistakes, and should easily access their understanding of the mathematics in order to correct them.

1. Perform arithmetic with signed numbers, including fractions and percentages.
2. Combine like terms in algebraic expressions.
3. Use the distributive law for monomials and binomials.
4. Factor monomials out of algebraic expressions.
5. Solve linear equations of one variable.
6. Solve quadratic equations of one variable.
7. Apply laws of exponents.
8. Plot points that are on the graph of a function.
9. Given the measures of two angles in a triangle, find the measure of the third.
10. Find areas of right triangles.
11. Find and use ratios from similar triangles.
12. Given the lengths of two sides of a right triangle, find the length of the third side.



Near the end of Appendix 4 there are sample mathematics questions to provide further explanation of these expectations. Now we would like you to be interactive with this chapter. Using the table on the next page, make some connections between one of your courses and the ICAS competencies. These competencies represent incoming college level competencies. If you teach a basic skills mathematics course you will be helping students to learn some of these depending upon the course level. How well can your students perform these competencies? Should you be assessing your student's ability to perform these skills? Are your students aware of the target – college level mathematics competencies? Think about a particular course you teach in mathematics. Check the appropriate boxes with respect to the ICAS mathematics competencies relevant to a course of your choice and consider whether this validates the curriculum and activities you use or indicates a potential area to include in your course expectations of students.

ICAS COMPETENCY	Relevant to my course	Included in my present curriculum	Not included in my curriculum
1. Perform arithmetic with signed numbers, including fractions and percentages.			
2. Combine like terms in algebraic expressions.			
3. Use the distributive law for monomials and binomials.			
4. Factor monomials out of algebraic expressions.			
5. Solve linear equations of one variable.			
6. Solve quadratic equations of one variable.			
7. Apply laws of exponents.			

ICAS COMPETENCY	Relevant to my course	Included in my present curriculum	Not included in my curriculum
8. Plot points that are on the graph of a function.			
9. Given the measures of two angles in a triangle, find the measure of the third.			
10. Find areas of right triangles			
11. Find and use ratios from similar triangles.			
12. Given the lengths of two sides of a right triangle, find the length of the third side.			

Are there any other important competencies that are crucial to your course which are not listed in the ICAS competencies? List them in the space below:

Another way to ensure alignment is to consider the work done between public education segments that address these alignment issues by creating common assessments. One example of this is Mathematics Diagnostic Testing Project (MDTP).

Statewide Intersegmental alignment - Mathematics Diagnostic Testing Project (MDTP)

The following information was taken directly from the MDTP website at <http://mdtp.ucsd.edu/>

MDTP is an intersegmental educational project in California that develops, distributes, scores, and reports the results of tests that measure student readiness for mathematics courses from first year algebra to calculus. MDTP provides scoring services for California's precollegiate schools and precollegiate outreach programs. MDTP materials may only be used outside of California by accredited educational institutions that have a current license with MDTP.

The CSU/UC Mathematics Diagnostic Testing Project was formed as a joint project of and supported by The California State University (CSU) and the University of California (UC) in 1977. The charge to the workgroup included determining mathematics areas in which

competency was necessary for success in certain mathematics courses and developing diagnostic tests over these areas. The tests were used by both university systems. By 1986, a series of four tests had been released for use by California high schools.

Every test that is released by MDTP is first field-tested, revised, and then field-tested again and, if necessary, revised again. One essential criterion to be satisfied by each test is that its topics and items are necessary for success in subsequent mathematics courses. The face validity of the tests' content is evidenced by the composition of the workgroup and by the widespread acceptance and use of the tests throughout California. Every campus of the University of California, approximately two-fifths of the campuses of The California State University, and approximately three-fifths of the campuses of the California Community Colleges use at least one of the MDTP tests as part of their matriculation process for entering students. In addition more than 6,600 secondary school teachers chose to administer MDTP tests to their students in 2001—2002.

Since 1982 MDTP has offered its tests and scoring services to California secondary schools. Individual diagnostic reports are provided for students as well as detailed item analyses and summary reports for teachers. The student reports indicate areas in which students did well and those areas in which the test results suggest a need for further study in order to be prepared for future coursework. The summary reports have been used by teachers to help identify areas of the curriculum that seem to be working well and other areas or topics where changes may be needed.

MDTP's primary goal is to help California's teachers prepare students for success in further study of mathematics by identifying strengths and weaknesses in their students' conceptual understanding and procedural skills.

More information about MDTP is available on the website at <http://mdtp.ucsd.edu/> and we have permission to include a sample written response and general scoring rubric for a sample question in Appendix 5.

Now that we know what defines college level mathematics skills, how do we incorporate that into our courses and programs overall? The first step in reaching the competencies, particularly any you indicated on the previous pages as including in your course, is to develop the student learning outcomes (SLOs) and assessments that correspond to the competency. Your college may already be actively involved in this effort, and your department may have already created these. But in case you are new to SLOs or just still confused, take a quick look at Chapter 15 of this handbook *Course Assessment Basics: Evaluating Your Construction* for a more detailed explanation about this process.

Mathematics faculty should discuss how these competencies can be converted into course level SLOs that then align with subsequent courses and create a final set of program SLOs for the department. Research in the Florida community colleges has revealed that the biggest barrier to student success in mathematics courses is a lack of alignment. Learning concepts in one mathematics class does not necessarily translate into success in the next class (Bachford & Slater, 2008, pp. 3, 11, 13).

One best practice that facilitates better alignment at some institutions is to assign each basic skills mathematics course a course coordinator and a common course final or at least a few embedded mathematics problems/questions within different course finals. The common course final or common mathematics problems/questions should be assessed under the direction of the course coordinator in a department meeting. The assessment should result in an action plan for changes in each course curriculum. Have a look below at the sample assessments and curricular discussions from San Diego Mesa College and Golden West College. What is that assessment based upon? How do we align basic skills courses to meet that target? And how do we keep students moving through the mathematics sequence to success? Below are some examples of SLOs and Assessments and conclusions some colleges have come to in order to improve the mathematics success.

B. Effective Practices: Creating a Mathematics Course that Focuses on Success Using Student Learning Outcomes

Course SLOs guide and direct the content covered in a course, its activities and assignments. In addition, the information gained from measuring them (called “assessments” in the SLO world) will give you valuable information about how your students are doing, and what you can do to improve their learning. This entire process can result in better success.

Below are some examples of SLOs from various California community colleges. Check out the website maintained by Larry Green of CMC³ that serves as a storehouse for all levels of Mathematics SLOs at <http://www.ltcconline.net/green1/SLO/SLOs/Math/MATSLOsStatewide.htm> Notice that colleges format their SLOs in different ways and emphasize different skill sets. The following are samples. Your SLOs should be consistent with your curriculum, relevant to your students and aligned to the subsequent course.

Pre-Algebra

College of San Mateo

1. Strengthen core entry skills, which are to perform
 - a. Operations with whole numbers
 - b. Operations with fractions
 - c. Operations with decimals
 - d. Operations with percentages
2. Perform operations on integers.
3. Simplify and evaluate variable expressions.
4. Solve a one variable first degree linear equation that models a situation.
5. Construct linear graphs.
6. Convert units of measure (includes American and Metric systems).
7. Perform operations with polynomial

Lake Tahoe Community College

Part I

1. Perform arithmetic operations with whole numbers, fractions, and decimals.
2. Translate written language into mathematical statements.
3. Apply the concepts in the course to real-life situations.

Part II

1. Solve problems involving decimals, percents and beginning algebra

2. Translate written statements in to mathematical statements.
3. Apply the topics of Basic Arithmetic (Part II) to real life situations.

Solano Community College

1. Perform arithmetic operations on signed values including integers, fractions, and decimals.
2. Simplify algebraic expressions, evaluate formulas, and solve basic linear equations and application problems.
3. Students obtain sufficient mathematics proficiency to be successful in a subsequent elementary algebra course.

Beginning Algebra College of San Mateo

1. Identify and apply basic algebraic concepts including slope, absolute value, scientific notation, equivalent equations, laws of exponents, intercepts, horizontal lines, and vertical lines.
2. Solve systems of linear equations in two unknowns using graphing, elimination, and substitution.
3. Solve equations and inequalities in one variable.
4. Solve quadratic equations by factoring and by using the quadratic formula.
5. Solve elementary radical equations.
6. Graph linear equations.
7. Solve problems by application of linear functions.
8. Apply the properties of and perform operations with radicals.
9. Apply the properties of and perform operations with integer exponents.

Lake Tahoe Community College

First Quarter

1. Solve linear equations and inequalities.
2. Define and employ terminology and arithmetic relating to polynomials in one variable.
3. Determine the equation and graph a line given information about the line.
4. Manipulate expressions with integral exponents.
5. Apply course topics to real-world situations.

Second Quarter

1. Factor a polynomial.
2. Apply the four basic operations to rational and radical expressions.
3. Solve equations with rational and radical expressions.
4. Solve a 2 x 2 system of linear equations.
5. Solve quadratic equations.
6. Apply course topics to real world situations.

Solano Community College

1. Distinguish between and give examples of equations, solutions to equations, and algebraic expressions.
2. Solve mathematical equations appropriate to the elementary algebra curriculum.
3. Formulate real-world problems quantitatively and interpret the results.

Coastline Mathematics 030 Intermediate Algebra.

1. Given a collection of data, determine if it fits a linear, quadratic, polynomial, rational, radical, exponential, logarithmic, or logistic function; find the curve of best fit; graph the model with the appropriate range and domain; analyze its behavior and make predictions.
2. Express a real-world problem as an equation or system of equations, estimate the answer, then solve the equation or system to find a solution and judge its reasonableness.
3. Identify and graph conic section equations including parabolas, circles, ellipses, and hyperbolas; show major and minor axes, intercepts, foci, and asymptotes; given a graph write the equation for the conic section in standard form; and solve applications involving conic sections.
5. Use appropriate technology such as calculators and/or computer software to solve mathematical problems, enhance mathematical thinking and understanding, and be able to judge the reasonableness of the results.
6. Communicate mathematically both verbally and in writing by explaining each step and justifying the answer after solving a problem.

C. Design a course with appropriate SLOs that align with subsequent courses Assessing SLOs: Creating Success



One of the ways that the SLO assessment process improves student success is that it asks faculty to write down the criteria that they use in grading. When students have a clear understanding of the criteria necessary to complete an assignment or exam, **they often do better**. Remember that students with basic skills needs are often particularly lacking in this knowledge, due to high school preparation (or the lack thereof) and previous academic experiences. Grading criteria needs to become more than the percentage of correct problems that result in a certain grade. What is the process you use in your head when you grade a problem? What are you looking for? Look at Appendix 6 for sample rubrics designed for specific mathematics assignments. These rubrics clearly describe the criteria expected by students, making grading easier and reducing student complaints and confusion.

Here's an example of what we mean. The SLOs in the mathematics class listed below indicate the ultimate grading criteria for this course. Students will not pass unless they can successfully accomplish these measurable outcomes. Faculty know if students are learning these essential competencies, the ones required in the SLOs, because all of the faculty teaching these courses use identical questions in their finals. The department then meets to discuss the data from the embedded questions to determine where they can improve instruction. Let's look at two completed SLO assessment cycles at two different colleges.

San Diego Mesa College: A Sample Assessment Practice and Plans to Improve Teaching

San Diego Mesa Mathematics department has agreed to embed questions on course finals as a departmental assessment. They've asked instructors to use the question(s) as part of their final OR give it as a separate quiz in or about finals week. The faculty do not all give the same final. Last semester, they had better than 50% response on getting the results back from the faculty, and this includes adjunct faculty teaching the same course. Although they encouraged all the instructors to respond, there is no way to enforce compliance.

Dealing with the Assessment Results

Most discussion of the results took place at a meeting of the algebra committee, with a short report at the department meeting. So far, any formal changes made have been to the question itself. This is not uncommon. Faculty usually do not discuss questions used for exams, and yet how many of us were trained in asking good questions? Many departments find that just the work in developing a common question provides valuable dialog and training.

Individual instructors report that giving a common assessment has affected their teaching in ways such as:

- they do more applied problems related to linear models in class
- they assign more such problems as homework
- they connect back to this idea in the chapter on functions

The general discussion among faculty has centered on two questions:

1. Is it important that students formally write out the model as opposed to calculating without using variables?

The consensus is generally, yes, because it is important for further work. As a result they have rephrased the question to emphasize using variables. Also, if the faculty concluded that if the students can do the problem without variables, it is not clear that they have actually taught them something new!

2. Should faculty also include relationships in which the independent variable is something other than time?

The discussion revealed that the text only has a few problems using something other than time, so this could be a factor in deciding to change texts. Alternatively, faculty suggested collaborating with the sciences to get some sample questions.

This is an example of what is meant by “closing the loop” in SLO assessment jargon. By embedding sample questions in departmental assessments faculty have created a learning environment to look at what they do, what students learn and consider how they can improve. The resulting discussions and changes made close the loop with the purpose of making students more successful. Many colleges report that discussing these issues, which faculty are often left to deal with on their own, has enlivened their teaching and improved the learning of their students.

Golden West College Mathematics Assessment and Plans for Improvement

Golden West College has provided this complete and more formalized Student Learning Outcomes Assessment Cycle report to use as another example.

Golden West College Mathematics Department Student Learning Outcomes For Mathematics 005

A. SLOs

Upon successful completion of Mathematics 005 the student shall be able to:

1. Know all the fundamental operations of arithmetic on whole numbers, fractions and decimals.
2. Do most of applications in percentage, ratio, proportion, and measurement
3. Converting between percentage, decimal and fraction
4. Reading graph, finding mean and median in statistics
5. Applying mathematics to real life application with mental mathematics calculation.

B. Embedded Test Questions

1. a) Evaluate: $(6^2 - (5 \cdot 5 + 2))^2$

b) Evaluate: $\sqrt{36} - \sqrt[3]{8}$

2. Kate invested 20,000 into an account that pays 7% annual interest compounded daily. Find the interest earned in this account after 5 years. (Round your answer to the nearest penny) **Given: compound interest factor is 1.41902**

3. Simplify: $(4 - 2) \cdot 6 \div 3 + (5 - 2)^2$

4. a) Find the LCM of 30 and 42. b) Find the GCF of 60 and 80.

5. A store marks up items by 28 percent. If an item cost the store \$20.25 from the supplier, what will the selling price be?

D. Collected Data

The following data represent the percentage of students who successively solved the embedded test question. The symbol * indicates less than 65% of the students were successful.

Question #1 90%

Question #2 83.5%

Question #3 99%

Question #4 75%

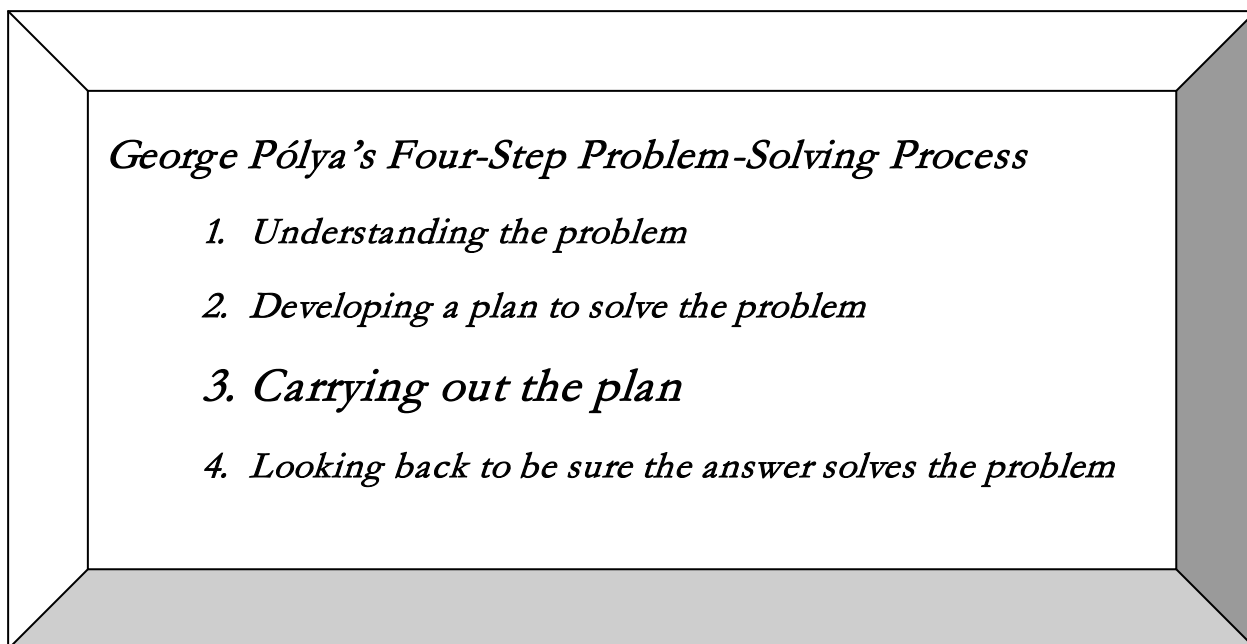
Question #5 57%

E. Recommendations for Improving Success Rates

1. Recommend that students seek early tutorial assistance.
2. Find and recommend quality websites on these difficult topics.

3. Prepare handouts to clarify difficult concepts.
4. Encourage students to use office hours.
5. Help students to form study groups.
6. Use, but with discretion, collaborative learning strategies.
7. Encourage students not to use a calculator with their homework.
8. Encourage students to practice mental mathematics calculation whenever they can in real life situation.

Now that you have had a chance to view some plans from other schools, let's look at Pólya's step 3: carrying out the plan.



3. Carrying out the plan

Plan Implementation Summary: Add at least one active learning strategy to each class session. Use SLOs to direct the activities, course content and to assess student understanding and abilities. Ultimately a course and program assessment will help to direct the entire process and promote student success.

During the Basic Skills Summer Institute in July 2008 we had mathematics faculty create brief descriptions of active learning lessons. These are included in Appendix 7. The faculty developed and taught these lessons and then, following effective practices, used a peer review rubric to assess the lesson. This modeled what our behavior in the class should be. However, many faculty are stuck on how to assess their teaching on the fly. When constructing your framework for basic skills success you want to get regular feedback during the time you are teaching. This next part of the chapter begins with samples of quick classroom assessment techniques, in order to monitor the learning in your classroom. Classroom assessment techniques allow you to adjust your pedagogy, but also to monitor learning on a real-time basis. We call this formative assessment because it helps you to

formulate and reformulate on the fly. It provides feedback so students become more responsive to the teaching and diagnostic information for the faculty member on what specific concepts the students need more time on. In order to give students specific and helpful information, faculty have found it useful to create rubrics detailing expectations. Sample rubrics are used throughout the chapter and in Appendix 5. After focusing on the curriculum and assessment, let's take a look at some of the pedagogical changes that have been shown to make improvements in success. The following quick and easy classroom assessment techniques will allow you to get immediate feedback on your students' expertise in what they are supposed to be learning.

One of the challenges of teaching mathematics at the community college level is that students have generally taken the class before. Some have attempted the class multiple times without success, leaving them with pockets of information that may be incorrect or incomplete. Determining which pieces of information they have, don't have or have processed incorrectly is part of assisting the student to be successful. Each of the following strategies assists - in addition to creating a classroom where the student is an active learner - in diagnosing the problem areas for individual students. As with cognitively guided instruction, the intent is to build on the mathematics knowledge of students based on what they already know.

Teaching is not a science; it is an art. If teaching were a science there would be a best way of teaching and everyone would have to teach like that. Since teaching is not a science, there is great latitude and much possibility for personal differences. ... let me tell you what my idea of teaching is. Perhaps the first point, which is widely accepted, is that teaching must be active, or rather active learning. ... the main point in mathematics teaching is to develop the tactics of problem solving. George Pólya (O'Connor & Robertson, 2002, p. 2)

Sample Mathematics Active Learning Activities

The following sample mathematics active learning activities -- Minute Paper, Muddiest Point, or on a 3 x 5 problem -- provide you with more opportunities to assess where the class is as a whole and the types of problems individual students are having. It is an opportunity to address individual weaknesses before the graded assessment.



Three by fives by Joan Córdova

At the beginning of the semester my students are asked to add 3x5 cards to their list of school supplies. Virtually every class session they are asked to use the 3x5 cards in a variety of classroom assessment exercises. The exercises vary depending on what is needed including Muddiest Point, Minute Papers, and sample problems.

Often the class will start with a 3x5 question on the board that assesses the topic of the previous lesson. It takes a few minutes for the students to work and less than a minute to read through the cards to see how well the class understood the lesson.

The cards are then shuffled and used to call on students throughout the class session. The cards keep me from unknown biases I may have when calling on students. Maybe I only call on the kids with pony tails or something. With the cards it allows me to call on them by name. They know their

card is in the stack so they have a possibility of being called on. By the end of the class I try to have made it all the way through the stack.

To help with the stress of being called on I use the “Who Wants to be a Millionaire” model. When a student is called on they have an opportunity for a life line or to poll the audience. What is interesting is that once they know they have an option, they will try the problem first. They are more willing to offer answers and opinions. This was a very interesting unexpected outcome. It also gives me an opportunity to ask “Is that your final answer” which gets them reviewing what they said.

Examples:

The first class after discussing addition of fractions with different denominators the 3 x 5 card at the beginning of class might ask: When adding fractions, what is the first thing you need to know? *Are the denominators the same?* is the correct response but the actual responses provide valuable insight.

When working on word problems the 3 x 5 card at the beginning of class might ask them to identify the steps taken to work a word problem. Or perhaps ask the students what the first step in solving a word problem could be. The answers may vary but what would be acceptable would be *Read the problem* or something along the lines of *Identify what the problem is asking.*



Minute Paper

Description:

No other technique has been used more often or by more college teachers than the *Minute Paper* (Angelo & Cross, 1993, p. 1). This technique -- also known as the *One-Minute Paper* and the *Half-Sheet Response* -- provides a quick and extremely simple way to collect written feedback on student learning. To use the *Minute Paper*, an instructor stops class two or three minutes early and asks students to respond briefly to some variation on the following two questions: "What was the most important thing you learned during this class?" and "What important question remains unanswered?" Students write their responses on index cards or half-sheets of scrap paper and hand them in.

Variations:

- What was the most important point of the section?
- What was the most surprising idea or concept?
- What question remains unanswered in your mind?
- What question from this class might appear on the next quiz/test?
- What was the muddiest point of the class?
- What was the main concept illustrated in class?

Step-by-Step Procedure:

1. Decide first what you want to focus on and, as a consequence, when to administer the *Minute Paper*. If you want to focus on students' understanding of a lecture, the last few minutes of class may be the best time. If your focus is on a prior homework assignment, however, the first few minutes may be more appropriate.

- Using the two basic questions from the "Description" above as starting points, write *Minute Paper* prompts that fit your course and students. Try out your *Minute Paper* on a colleague or teaching assistant before using it in class.
- Plan to set aside five to ten minutes of your next class to use the technique, as well as time later to discuss the results.
- Before class, write one or, at the most, two *Minute Paper* questions on the chalkboard or prepare an overhead transparency.
- At a convenient time, hand out index cards or half-sheets of scrap paper.
- Unless there is a very good reason to know who wrote what, direct students to leave their names off the papers or cards.
- Let the students know how much time they will have (two to five minutes per question is usually enough), what kinds of answers you want (words, phrases, or short sentences), and when they can expect your feedback.

Use this Mathematics Example from Joan Córdova for a quick classroom assessment

After presenting information on simplifying fractions in an algebra class, the next class session might begin using a 3 x 5 problem such as:

Simplify:

$$\frac{x^2 - x}{x^2}$$

When the cards are turned in they are quickly scanned for correct responses and common errors, failing to factor the numerator before removing factors of 1, canceling the x^2 terms, getting as far as

$$\frac{x(x-1)}{x^2}$$

or

$$\frac{x-1}{x}$$

and then canceling willy nilly (that is a technical term for random removal of terms) rather than removing factors of 1.

The 3 x 5 is then reviewed discussing correct procedure and errors found on the cards calling on students for details on factoring, factors of 1 etc.



Muddiest Point

Description:

The *Muddiest Point* is just about the simplest technique one can use (Angelo & Cross, 1993, p. 1). It is also remarkably efficient, since it provides a high information return for a very low investment of time and energy. The technique consists of asking students to jot down a quick response to one

question: "What was the muddiest point in?" The focus of the *Muddiest Point* assessment might be a sample problem, a lecture, a discussion, or a homework assignment.

Step-by-Step Procedure:

1. Determine what you want feedback on: the entire class session or one self-contained segment? A lecture, a problem, a discussion?
2. If you are using the technique in class, reserve a few minutes at the end of the class session. Leave enough time to ask the question, to allow students to respond, and to collect their responses by the usual ending time.
3. Let students know beforehand how much time they will have to respond and what use you will make of their responses.
4. Pass out slips of paper or index cards for students to write on.
5. Collect the responses as or before students leave. Stationing yourself at the door and collecting "muddy points" as students file out is one way; leaving a "muddy point" collection box by the exit is another.
6. Respond to the students' feedback during the next class meeting or as soon as possible afterward.

Here is another example from Joan Córdova:

Following a demonstration on adding fractions with different denominators, leaving the last problem (with vocabulary) on the board, ask the students to take four minutes to write down the step in the problem that is the muddiest point for them. Ask them to specify the difficulty, if they can.

$$\frac{3}{5} + \frac{7}{10}$$

$$5, 10 \quad \text{LCD} = 10$$

$$\frac{3*2}{5*2} + \frac{7}{10}$$

$$\frac{6}{10} + \frac{7}{10}$$

$$\frac{6+7}{10}$$

$$\frac{13}{10}$$

The fractions have different denominators

Find the lowest common denominator by factoring the denominators

Find equivalent fractions by multiplying by factors of one.

Combine numerators, keep the denominator.

Each line of work is written in a different color ink which assists the students in identifying the line where they don't understand the work. What was done is written out to assist the students in understanding what is being done.

When students have particular difficulty with topics they are asked to work problems in three columns. The first column is their computation, the second column is the description of the computation, the third column is the 'rule' that allows the step.



Raise your hand with a twist by Joan Córdova

There are times in the middle of a class when it is informative to know whether the students think they have grasped the concept. One very quick assessment of this is having them raise their hand – with a twist. Their hand is a scale from 0 – 5, representing where they are on the topic being discussed. It is a quick read of where they are and allows for some help from classmates if there are a few students who are unclear on the concept.

Asking the students to give me a “read” of where they are on the topic involves two steps. The first is to determine a scale. The second is to then scan the room to see how they respond. It takes less than a minute of class time.

If I have just finished working a problem and want to know if the students need to see another example or are they ready to go on, the rubric might be

Fist	What? Are you talking to me? What class is this?
1 finger up	I'm beginning to see a type of problem.
2 fingers up	With a tutor and a lot of time I could work the problem.
3 fingers up	It might take some assistance for me to work the problem.
4 fingers up	I could work these types of problems with help from my notes.
5 fingers up	I could work this type of problem on my own.

Or:

Fist	If you don't know what the main topic is.
1 finger up	If you know the topic.
2 fingers up	If you know the topic and one subtopic.
3 fingers up	If you know the topic and two subtopics.
4 fingers up	If you know the topic and three subtopics.
5 fingers up	If you know the topic and most of the subtopics.

The scale can be less defined by asking the students how confident they feel with the topic with the fist being not at all and the 5 fingers being ready to take the test.

By the middle of the semester the students are making the assessment on their own. Some will flash between two decisions. 3 - 4 fingers or 4 – 5 fingers indicating they are almost to the next step but not quite there yet.

It can also be used to determine the depth of their knowledge. For example at the end of a lesson on quadratics asking them to show how many ways they know how to solve a quadratic equation.

Since we usually discuss four options their responses would be from fist to four fingers. For additional information you can ask a follow up question, of the number of ways you know how to solve a quadratic equation, how many are you comfortable using? How many would you be able to list the steps involved in the solution?

Contextualized Mathematics Applications

Teaching mathematics in context is an important and effective technique for dealing with basic mathematics skills. We know that not all mathematics education takes place in mathematics classes. Contextualized mathematics application involves embedding relevant mathematics lessons in a discipline topic in a non-mathematics course. For instance, focusing on the math skills necessary in an automotive course or in a biology experiment make that mathematics more real and engage the student.

Focusing on mathematics in other courses is an essential strategy to creating value in mathematics skills. Career technical courses are trying to create contextual lessons in vocational courses that enable students to relearn or cement those basic mathematics skills. Mathematics faculty need to work with other disciplines to develop interesting assignments and activities that show the relevance of basic mathematics skills in many contexts. Take a look at the contextualized mathematics assignments found in Appendix 8 as examples.

Contextualizing mathematics in other disciplines areas means that mathematics faculty will need to collaborate with faculty in other disciplines to create lessons that are accurate and complete. We have been told that these collaborations have yielded benefit to all faculty involved. Mathematics faculty often gain insight into authentic math applications they can build into their mathematics courses as well as helping the discipline faculty.

MAPS - A Program that Creates Mathematics Success

One other approach to enhancing success for basics skills students is creating programs which combine student services with mathematics. One such program is the MAPS program at Mission College, which serves a diverse group of students, recruited from several Mission College programs, including EOPS, Access, Avanzar, and DISC. In addition, the program actively seeks to include students from those groups who have traditionally had poor success in basic skills and college mathematics courses.

A counselor is available for each class section. The counselor and instructor work closely to ensure student success. The counselor is available daily during class to talk to students regarding their grade to date, missing assignments and absences. In addition, the counselor teaches study skills and provides individual and academic counseling for students in the program. The MAPS team of instructors and counselor meet on a weekly basis to plan program activities and discuss concerns related to students' achievement in the class.

In addition to in-class tutoring, the program offers students group tutoring outside of class. Each week, approximately 10 hours of tutoring are offered at various times throughout the day and early

evening. The tutors are trained to reinforce the methods and approach taught in regular class. For students interested in working with other students outside of class, study groups have also been formed. Whenever possible, a tutor attends the study group to assist students with questions.

The program also arranges for guest speakers to visit the classes. These speakers have included men and women working in technical field, motivational speakers, and informational sessions on transfer agreements to the UC or CSU system.

MAPS Services

- * Academic, career, and personal counseling
- * Books on loan provided by Title V
- * Extended instruction time
- * Personalized attention and support from teacher, tutors, counselor, and peers
- * Tutoring in and outside of class
- * Improved study skills
- * Guest speakers
- * Participation in a class community

You can find more information on their website at

<http://www.missioncollege.org/Depts/Math/MAPS/index.html> or by contacting Linda Retterath - linda_retterath@wvm.edu

Pasadena City College Summer Bridges and Math Jam

The Pasadena City College Summer Bridges and Math Jam programs are integrated approaches that combine multiple efforts. The special summer programs--bridges (basic skills-level classes combined with student success strategies) and "jams" (Two-week study jams in mathematics combined with student success strategies) work with Faculty Inquiry Groups (FIGs) to better prepare teachers to address the needs of basic skills students, revise curriculum, adapt pedagogy, etc.

FIGs ask faculty to become researchers of basic skills students and basic skills teaching and learning. The programs engage faculty to gain a deeper level of understanding of teaching and learning, which transform attitudes and practices to improve student success. The faculty have identified many problems such as:

- Word problems are hard: students avoid them and teachers struggle to teach them.
- There is too much material to cover in class.
- New concepts in the last chapters are rushed through because faculty run out of time.
- It is hard to find time to show students real-world applicability.

After identifying these problems faculty worked to create more effective strategies to address them. You can check out the results at Chung's website, called *Not Lost in Translation: How Yu-Chung helps her students understand (and love) word problems* found at

http://gallery.carnegiefoundation.org/specc/specc/specc_homepage.html

In addition, faculty reduced the number of topics to a reasonable number that could be covered, learned and retained by students. Some faculty moved the concepts at the end of the book to the beginning of the term. They determined that these lessons were important and so re-prioritized the

order of the topics covered. Finally, faculty developed interactive hands-on approaches that embraced real-world applicability and incorporated these into their lessons, much to the delight of their students. This research centered approach to examining and revising curriculum has created greater success for the students, more fulfillment for the faculty, and a habit of using inquiry to identify and solve teaching challenges.

Reading and Mathematics by Dianne McKay

Reading in mathematics requires a specific approach. Mathematics reading involves deciphering material that is factually very dense and has its own vocabulary that sometimes students find as difficult as learning a foreign language. It also often requires visualizing as one reads. In addition, reading and understanding word problems involves the higher level thinking skills of application, analyzing, and synthesizing, in addition to literal comprehension.

Reading in mathematics requires students to SLOW way down, **reading at approximately 10% of their normal reading rate**, pulling the meaning out of every word, phrase and sentence, and testing understanding before moving on. Students should sketch graphs, study sample problems, and refer back to explanations as they work through homework problems in a chapter.

Mathematics, like learning a foreign language, is progressive. What a student doesn't learn in Chapter One will haunt them in Chapter 2, and by Chapter 3 they are often lost. Students need help not only to read mathematics, but also in management skills that teach them to keep current in order to be successful.

Sierra College teaches a workshop on how to read word problems to help students with a major hurdle in mathematics reading. What follows is a description of their program.

Sierra College's Reading and Mathematics Connections

When developmental students have difficulty with reading, they can also have trouble with many other subjects they take while in college. In fact, many educators agree that reading lies at the very heart of all other disciplines. All courses have some component of reading imbedded in their content, so it is logical to assume that the inability to effectively decode the written word can greatly hamper the learning process for students.

At Sierra College, mathematics professors agree that reading is an integral part of students' ability to solve word problems. Many of these same professors agree that the computational part is not what causes student difficulty with word problems. In fact, if the problem itself is set up for students without them having to read and decode it, they are able to complete the mathematics portion of the problem. The difficulty arises when students don't read the problem carefully and cannot visualize or interpret the words so that they know what it is they are looking for. That is why the steps of pacing, annotating, translating, and paraphrasing are important to finding success with word problems.

As a means of helping students understand the process of decoding mathematics word problems, Sierra College offers a free ninety-minute Student Success Workshop nearly every semester. Students can attend this workshop and learn a great deal about the process of understanding the reading of

word problems, and they are given specific tools to help them turn the written word into a mathematical equation. Below is an explanation of the process mathematics and reading faculty use to teach this workshop as well as handouts they use with students during the workshop itself. Sara Pries, Sierra College mathematics instructor explains the workshop process.

Word Problems Made Easy

I open by talking about word problems being puzzles that need to be taken apart and put back together in a logical sequence. Then each student is given five puzzle pieces that they must put together to form a square. I then talk to them about the importance of understanding what they are reading and of having a logical approach to solving any word problem. I stress that the workshop will not focus on the actual answers but on the process. Then I hand out **“Seven Reading Problems”** (See Appendix 9) for them to do. For example: Is it legal in North Carolina for a man to marry his widow’s sister?

Lisa Rochford, Reading Instructor, then hands out and discusses **“How to Annotate a Text”** and **“Word Problems Step-by-Step”** that go through pacing, annotating, translating and paraphrasing. Then we give them **“Five Word Problems: Practicing Pace, Annotate, Translate, and Paraphrase”** (see Appendix 9) and model how to use what she has just gone over. We write variable statements and equations for each one but do not work out the equations. We stress being able to sort out what it is they are trying to find (the main idea) and what the supporting details (facts) are.

Lynn Hargrove, mathematics instructor, then hands out and discusses key words and other information on **“Math Facts Information,”** and she goes over **“Tips for Solving Word Problems Involving Multiplication and Division of Fractions,”** **“Solving Application Problems,”** and the **“Three-Step Problem Solving Procedure”** (See Appendix 9).

CRLA Certification by Wade Ellis

Many mathematics faculty have discovered that the problem with mathematics for many students is the inability to read. The College Reading and Learning Association (CRLA) certifies a college’s tutor training program. At this time, there are over 500 colleges and universities that have this certification. There are three levels of certification: regular, advanced, master.

Each level requires an additional 10 hours of training and 25 hours of experience. After meeting the requirements for an initial institutional certification for one year, CRLA offers a three-year renewal certification which can be followed by a five-year recertification.

Tutors that have gone through such training can provide better tutorial assistance that emphasizes strategies and processes over simple content mastery. Such programs help tutors understand the need for tutees to be responsible for their own learning and begin to self-develop their own learning skills. Such certified tutorial training programs also help Tutorial Centers move toward process orientation and greater collaboration with instructors and course student learning outcomes.

For more information on how you can work on reading in your mathematics class, look at Chapter 10 of this handbook in the reading and mathematics section.



Directed Learning Activities by Wade Ellis

A Directed Learning Activity (DLA) is a flexible learning tool used by a college to integrate a Tutorial Center into the mathematics (and other discipline) curricula with the added bonus that hours by arrangement can be legitimately collected from the Chancellor's Office. An instructor decides on a mathematical activity tied to the course curriculum that may:

- review a concept or skill before it is needed in class,
- enhance a student learning skill, or
- build toward proficiency in a specific student learning outcome (SLO).

The instructor constructs the DLA according to well-developed instructional design criteria, provides the Tutorial Center with the written material for the DLA, and the Tutorial Center in conjunction with the instructor trains the tutors who will help the student assess their work on the DLA in the Tutorial Center. The course syllabus will require eight or more such DLAs during the course.

The use of DLAs increases student use of the Tutorial Center for tutoring beyond the DLAs, provides funding for the Tutorial Center, integrates supplemental instruction closely with courses, and can provide a method for increasing student performance on the measurable course SLOs.

Chaffey College has successfully implemented a DLAs program in their Mathematics Success Center.

Instruction Design Criteria

- The language of the activity clearly connects to the course assignments, objectives, and/or outcomes.
- The tutor and Tutorial Center are an essential component of the activity.
- The classroom instructor's directions guide a significant portion of the activity. The tutor mainly functions to review and enhance the learning experience.
- The activity clearly indicates how the outcome will be further developed through classroom instruction.
- The goal of the activity is the development of skills and strategies rather than the mere completion of exercises.



Using Rubrics to Clarify Expectations for Work in Mathematics and Enhance Success

For all assessments, it is important to clearly state what you expect and then to link those expectations to the Student Learning Outcomes. Many mathematics faculty have accomplished detailing criteria through writing rubrics that explain each aspect of the assignment. Creating a rubric to grade work becomes essential when using common exams or embedded questions. This allows the faculty to judge the work by the same standards.

As an example, take a look at the following mathematics rubrics, from the Barstow College Academic Skills mathematics courses. These are used to explain scoring for extended responses to mathematical assignments and provide specific diagnostic help to the student. The very useful and innovative aspect of these rubrics is that a student rubric is provided in order to stimulate self-evaluative thinking concerning the student's own mathematics abilities. Barstow has found that using these two rubrics in combination has resulted in better student success overall. In addition, faculty can use these rubrics and their results to assess course SLOs for accreditation using a process called course embedded assessment (Again, look at Chapter 15 of this handbook). The beauty of this process is that information is provided to the student to help him or her learn and information is revealed to faculty, helping him or her to teach better. We have collected a variety of mathematics rubrics for a variety of assignments. They begin under Appendix 6.

MATHEMATICS SCORING RUBRIC: A GUIDE TO SCORING EXTENDED-RESPONSE ITEMS

	MATHEMATICAL KNOWLEDGE: Knowledge of mathematical principles and concepts which result in a correct solution to a problem.	STRATEGIC KNOWLEDGE: Identification and use of important elements of the problem that represent and integrate concepts which yield the solution (e.g., models, diagrams, symbols, algorithms).	EXPLANATION: Written explanation of the rationales and steps of the solution process. A justification of each step is provided. Though important, the length of the response, grammar, and syntax are not the critical elements of this dimension.
Score Level 4	<ul style="list-style-type: none"> ◆ shows complete understanding of the problem’s mathematical concepts and principles ◆ uses appropriate mathematical terminology and notations including labeling answer if appropriate ◆ executes algorithms and computations completely and correctly 	<ul style="list-style-type: none"> ◆ identifies all important elements of the problem <u>and</u> shows complete understanding of the relationships among elements ◆ shows complete evidence of an appropriate strategy that would correctly solve the problem 	<ul style="list-style-type: none"> ◆ gives a complete written explanation of the solution process; clearly explains <u>what</u> was done and <u>why</u> it was done ◆ may include a diagram with a complete explanation of all its elements
3	<ul style="list-style-type: none"> ◆ shows nearly complete understanding of the problem’s mathematical concepts and principles ◆ uses mostly correct mathematical terminology and notations ◆ executes algorithms completely; computations are generally correct but may contain minor errors 	<ul style="list-style-type: none"> ◆ identifies most important elements of the problem and shows a general understanding of the relationships among them ◆ shows nearly complete evidence of an appropriate strategy for solving the problem 	<ul style="list-style-type: none"> ◆ gives a nearly complete written explanation of the solution process; clearly explains <u>what</u> was done and begins to address <u>why</u> it was done ◆ may include a diagram with most of its elements explained
2	<ul style="list-style-type: none"> ◆ shows some understanding of the problem’s mathematical concepts and principles ◆ uses some correct mathematical terminology and notations ◆ may contain major algorithmic or computational errors 	<ul style="list-style-type: none"> ◆ identifies some important elements of the problem but shows only limited understanding of the relationships among them ◆ shows some evidence of a strategy for solving the problem 	<ul style="list-style-type: none"> ◆ gives some written explanation of the solution process; either explains <u>what</u> was done or addresses <u>why</u> it was done ◆ explanation is vague, difficult to interpret, or does not completely match the solution process ◆ may include a diagram with some of its elements explained
1	<ul style="list-style-type: none"> ◆ shows limited to no understanding of the problem’s mathematical concepts and principles ◆ may misuse or fail to use mathematical terminology and notations ◆ attempts an answer 	<ul style="list-style-type: none"> ◆ fails to identify important elements or places too much emphasis on unrelated elements ◆ reflects an inappropriate strategy for solving the problem; strategy may be difficult to identify 	<ul style="list-style-type: none"> ◆ gives minimal written explanation of the solution process; may fail to explain <u>what</u> was done and <u>why</u> it was done ◆ explanation does not match presented solution process ◆ may include minimal discussion of the elements in a diagram; explanation of significant elements is unclear
0	<ul style="list-style-type: none"> ◆ no answer attempted 	<ul style="list-style-type: none"> ◆ no apparent strategy 	<ul style="list-style-type: none"> ◆ no written explanation of the solution process is provided

August 2005

“STUDENT-FRIENDLY” MATHEMATICS SCORING RUBRIC

Score Level (How many points do you earn?)	MATHEMATICAL KNOWLEDGE: (Do you know it?)	STRATEGIC KNOWLEDGE: (How do you plan?)	EXPLANATION: (Can you explain it?)
<p align="center">4</p>	<ul style="list-style-type: none"> ◆ I get the right answer, and I label it correctly. ◆ I use math terms correctly to show I understand how math works. ◆ I compute with no errors. 	<ul style="list-style-type: none"> ◆ I find all the important parts of the problem, and I know how they go together. ◆ I show all the steps I use to solve the problem. ◆ I explain any work I do in my head or with a calculator. ◆ I completely show pictures, diagrams, models or computation if I use them in my plan. 	<ul style="list-style-type: none"> ◆ I write <u>what</u> I did and <u>why</u> I did it in a clear and concise manner. ◆ If I use a drawing, I can explain all of it in writing. ◆ I describe my logical steps and my critical thinking in a step-by-step fashion.
<p align="center">3</p>	<ul style="list-style-type: none"> ◆ I use most math terms correctly and my answer is reasonable. ◆ I make minor errors in computation. ◆ I understand my mistake 	<ul style="list-style-type: none"> ◆ I find most of the important parts of the problem. ◆ I show a reasonable plan and most of the steps I use to solve the problem. 	<ul style="list-style-type: none"> ◆ I write mostly about <u>what</u> I did and not <u>why</u> I did it. ◆ I describe my steps, but not clearly. ◆ If I use a drawing, I can explain most of it in writing.
<p align="center">2</p>	<ul style="list-style-type: none"> ◆ I know how to do parts of the problem, but I make major errors in computation and get a wrong answer. ◆ I give a wrong answer or only part of the answer. 	<ul style="list-style-type: none"> ◆ I find some of the important parts of the problem. ◆ I show some of the steps, but my plan is not clear. 	<ul style="list-style-type: none"> ◆ I write some about <u>what</u> I did or <u>why</u> I did it but not both. ◆ If I use a drawing, I can explain some of it in writing.
<p align="center">1</p>	<ul style="list-style-type: none"> ◆ I try to do the problem, but I don't understand it. ◆ My answer is wrong and I cannot explain why. 	<ul style="list-style-type: none"> ◆ I find some parts of the problem. ◆ I show a plan that is limited. ◆ I show a limited number of steps I use to solve the problem. ◆ I may include unnecessary information. 	<ul style="list-style-type: none"> ◆ I write or draw something that doesn't go with my answer. ◆ I write an answer that is not clear.
<p align="center">0</p>	<ul style="list-style-type: none"> ◆ I don't try to answer the problem. 	<ul style="list-style-type: none"> ◆ I don't show a plan. 	<ul style="list-style-type: none"> ◆ I don't explain anything in writing.

August 2007

George Pólya's Four-Step Problem-Solving Process

- 1. Understanding the problem*
- 2. Developing a plan to solve the problem*
- 3. Carrying out the plan*
- 4. Looking back to be sure the answer solves the problem*

4. Assessing to be sure the answers (active learning, smaller class size, contextualized mathematics, clear student learning outcomes, evaluation using criteria in rubrics, etc) solves the problem

None of the strategies or exercises we have discussed in this chapter have any value UNLESS we solve our problem of improving low student success in basic skills mathematics.

Summary

- 1. Understanding the problem** - Postsecondary students were more likely to enroll in a remedial mathematics than in a remedial reading or writing class. The failure rate in these courses is alarming. Fewer than one-half are successful on their first attempt. Remember the problem was that basic skills mathematics success was very low, 48%, and acted as a barrier to student success in college overall.
- 2. Developing a plan to solve the problem** – Research shows class size plus active learning are effective indicators of success. The plan was three fold: 1) implement active and contextualized learning strategies in classes with appropriate class sizes, 2) clearly communicate the college level competencies including student's own responsibilities in mathematics classes, and 3) design a course around appropriate SLOs that align with subsequent courses.
- 3. Carrying out the plan** – Add one active learning strategy to each class session. Using SLOs to direct the activities and course content student understanding and abilities need to be assessed. Several quick classroom assessment techniques were described and then rubrics were shared to assess course work. Ultimately a course and program assessment help to direct the entire process.

4. **Looking back to be sure the answer solves the problem** – Review your class outcomes. Did more students complete the class? Did more students pass the class? Review the successful programs presented in the chapter. Are there additional changes to be made to increase student involvement in the class? So how do we know if these are effective strategies? Again the answer is a type of assessment. It is assessment at a higher level, the real professional level of teaching. We look at the outcomes, examine data, and disaggregate it to look for patterns. It is referred to as closing the assessment loop.

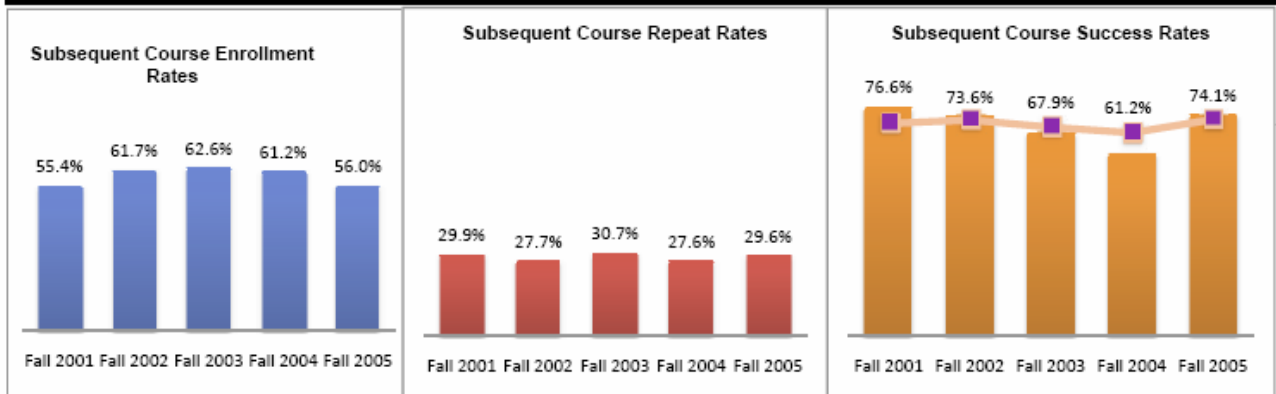
What follows is data collected and shared with us by Skyline College on their basic skills mathematics course success, progression, and success in college level courses. It has been disaggregated by gender, age and ethnicity. Several colleges have done this and, while it varies a bit from college to college, the trends obvious in these data are similar throughout the state and would most likely ring true in your institutional data. Recently, Skyline began to implement SLOs, assessment and a basic skills strategic plan based on these outcomes. These data will form a baseline to determine whether they have solved the problem identified in the research. This model of “looking back” to see trends in longitudinal data represents an evidence-based type of inquiry that has been useful in transforming problems of this nature.

The strategies Skyline College employs to address basic skills mathematics success will need to address the needs of students in different ways and most definitely ask the question “Are some student populations succeeding at far lower rates?” What considerations (cultural, curricular, programmatic, etc.) will make success achievable by more students? Look at the data on the next page and list the questions that come to your mind. Now that Skyline College has baseline data, what should they measure in the future? What pedagogical changes might help students to succeed?

Now read Appendix 2 Pathways through Algebra. This study provides a completed cycle of inquiry, data and identification of problems, analysis of interventions and recommendations to improve. Become part of the group of innovative faculty who believe, based on data, that if students work with us they can learn anything – even MATH!

SKYLINE COLLEGE
Subsequent Course Enrollment, Repeat and Success Over Two Years
Math Course Sequence
Overall

Term	Math 120 Math 122/123	Math 130, 150, 200, 201 or 241						None Cohort Course Success
	Cohort	Subsequent Course Enrollment		Subsequent Course Repeat		Subsequent Course Success		
		Cnt	Pcnt	Cnt	Pcnt	Cnt	Pcnt	
Fall 2001	193	107	55.4%	32	29.9%	82	76.6%	71.18%
Fall 2002	240	148	61.7%	41	27.7%	109	73.6%	72.42%
Fall 2003	219	137	62.6%	42	30.7%	93	67.9%	69.06%
Fall 2004	219	134	61.2%	37	27.6%	82	61.2%	68.07%
Fall 2005	193	108	56.0%	32	29.6%	80	74.1%	72.56%
Total/Average	1064	634	59.6%	184	29.0%	446	70.3%	70.80%



Source: SMCCCD Data Warehouse

Cohort: Number of transfer, degree, certificate seeking and undecided students in each fall term who enrolled and successfully completed Math 120 or Math 122/123 with a grade of A, B, C or CR.

Subsequent Course Enrollment: Number of students who successfully completed Math 120 or Math 122/123 and subsequently enrolled in either Math 130, 150, 200, 201 or 241 during a five term period.

Subsequent Course Repeat: Number of students who repeated Math 130, 150, 200, 201 or 241 one or more times during a four term period.

Subsequent Course Success: Number of students who subsequently enrolled in Math 130, 150, 200, 201 or 241 and received an A, B, C or CR grade notation during a five term period. Students may have repeated the same transfer course one or more times during a four term period but only the highest grade is used to calculate the course success rate.

None Cohort Course Success: Percent of none basic skills students who enrolled and successfully completed Math 130, 150, 200, 201 or 241 during a five term period. None basic skills students may have repeated the same transfer course one or more times during a four term period but only the highest grade is used to calculate the course success rate.

Office of Planning, Research and Institutional Effectiveness



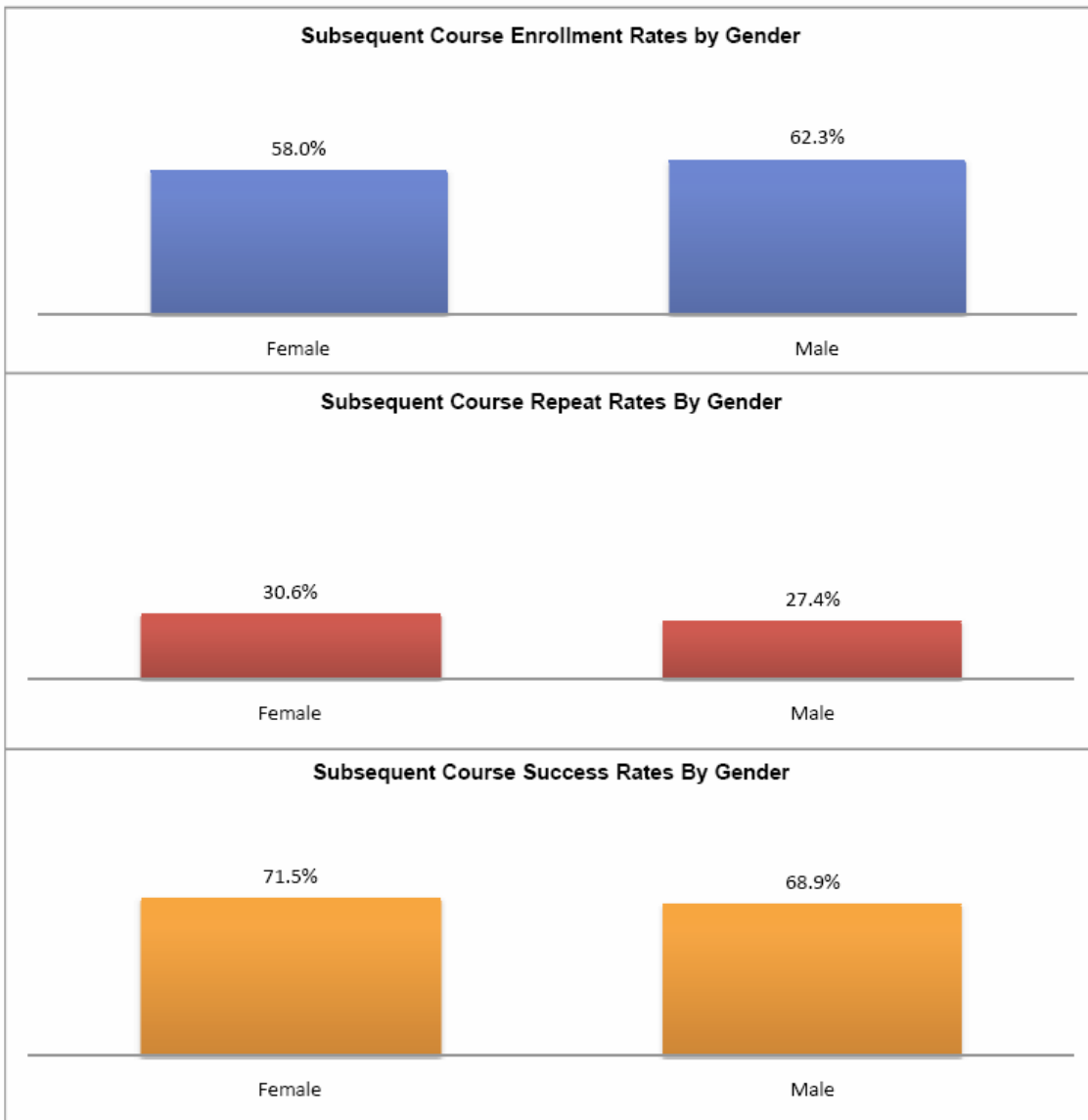
Source: SMCCCD Data Warehouse

Cohort: Number of transfer, degree, certificate seeking and undecided students in each fall term who enrolled and successfully completed Math 120 or Math 122/123 with a grade of A, B, C or CR.

Subsequent Course Enrollment: Number of students who successfully completed Math 120 or Math 122/123 and subsequently enrolled in either Math 130, 150, 200, 201 or 241 during a five term period.

Subsequent Course Repeat: Number of students who repeated Math 130, 150, 200, 201 or 241 one or more times during a four term period.

Subsequent Course Success: Number of students who subsequently enrolled in Math 130, 150, 200, 201 or 241 and received an A, B, C or CR grade notation during a five term period. Students may have repeated the same transfer course one or more times during a four term period but only the highest grade is used to calculate the course success rate.



Source: SMCCCD Data Warehouse

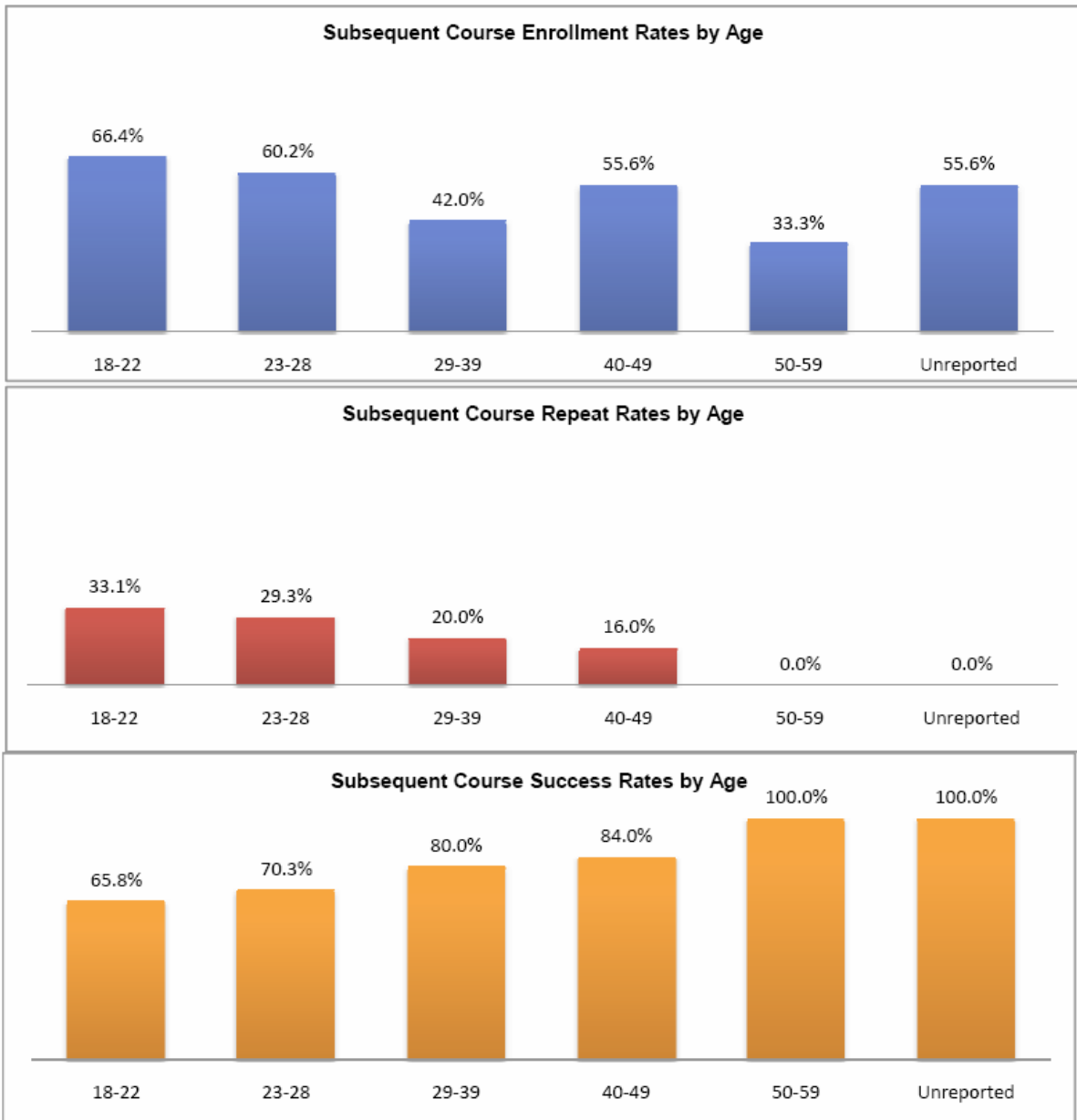
Note: "Unreported" students not included.

Cohort: Number of transfer, degree, certificate seeking and undecided students in each fall term who enrolled and successfully completed Math 120 or Math 122/123 with a grade of A, B, C or CR.

Subsequent Course Enrollment: Number of students who successfully completed Math 120 or Math 122/123 and subsequently enrolled in either Math 130, 150, 200, 201 or 241 during a five term period.

Subsequent Course Repeat: Number of students who repeated Math 130, 150, 200, 201 or 241 one or more times during a four term period.

Subsequent Course Success: Number of students who subsequently enrolled in Math 130, 150, 200, 201 or 241 and received an A, B, C or CR grade notation during a five term period. Students may have repeated the same transfer course one or more times during a four term period but only the highest grade is used to calculate the course success rate.



Source: SMCCCD Data Warehouse

Note: Students age 15 to 17 and 60+ are not included.

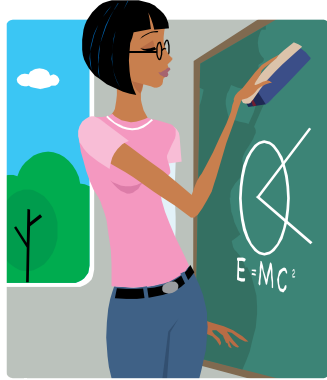
Cohort: Number of transfer, degree, certificate seeking and undecided students in each fall term who enrolled and successfully completed Math 120 or Math 122/123 with a grade of A, B, C or CR.

Subsequent Course Enrollment: Number of students who successfully completed Math 120 or Math 122/123 and subsequently enrolled in either Math 130, 150, 200, 201 or 241 during a five term period.

Subsequent Course Repeat: Number of students who repeated Math 130, 150, 200, 201 or 241 one or more times during a four term period.

Subsequent Course Success: Number of students who subsequently enrolled in Math 130, 150, 200, 201 or 241 and received an A, B, C or CR grade notation during a five term period. Students may have repeated the same transfer course one or more times during a four term period but only the highest grade is used to calculate the course success rate.

Appendixes
Chapter 9
Effective Practices in Mathematics: Specialty Supplies



Appendix 1: Quiz Answers

Appendix 2: Pathways through Algebra Project: Improving Student Success Rates in Elementary Algebra

Appendix 3: Mathematics Study Skills Inventory

Appendix 4: ICAS Mathematics Competencies and sample problems

Appendix 5: General Scoring Rubric for Written Response Items (MDTP Project)

Appendix 6: Sample Rubrics

Appendix 7: Sample Interactive Mathematics Lessons

Appendix 8: Contextualized Mathematics Activities

Appendix 9: Reading and Mathematics resources

Appendix 10: Resources for Chapter 9

Appendix 1

Quiz Answers

1. Students with basic skills needs are more likely to sign up for which of the following courses?
B. Pre-collegiate or basic skills mathematics courses
2. The California Community College success rate (meaning an A,B,C, or pass) for students in pre-collegiate or basic skills mathematics courses in Spring 2007 was approximately
C. 45 – 49% (47.6 for Basic Skills students available at <http://www.cccco.edu/SystemOffice/Divisions/TechResearchInfo/MIS/DataMartandReports/tabid/282/Default.aspx>)
3. Select the equation that best represents the relationship of the success rates in the following basic skills courses?
C. the highest success rate is in basic skills ESL then English and finally mathematics with the lowest success rate
4. Which of the following strategies have been shown by research to contribute to success in basic skills mathematics courses?
E. ALL of these produce higher success; contextualizing the mathematics to real world applications, smaller class size associated with feedback and interaction, active learning strategies where students must facilitate their own learning, high expectations for student responsibility
5. The ICAS (Intersegmental Committee of Academic Senates) mathematics competencies
B. are the skills that students must routinely exercise without hesitation in order to be prepared for college work as agreed upon by the CC's, CSU's and UC faculty
6. What percent of students who assess into basic skills mathematics three levels below college ever make it to college level mathematics?
A. Less than 10%
7. Which of the following have occurred where student learning outcomes have been created for mathematics classes?
E. all of the above; Faculty, both full-time and part-time, are focused on the same outcomes but are free to use their own techniques to help student gain mathematics skills, Courses have been aligned within a mathematics sequence, mathematics pre-requisites can be more easily validated for non-mathematics courses requiring pre-requisite mathematics knowledge, Program review and program level assessments have been simplified.
8. Research has shown that students' ability in mathematics is often
D. very dependent upon students' literacy or reading skills
9. Which of the following are true of rubrics?
E. Rubrics define criteria and expectations for student work allowing the student to evaluate his or her own work and the faculty member to grade more uniformly
10. Student success in basic skills mathematics courses can be improved and has been exemplified in several California Community College research programs.
a. True

Appendix 2

Pathways through Algebra Project

Improving Student Success Rates in Elementary Algebra

Kenneth Meehan, Ph.D. and Hal Huntsman

Retrieved from http://www.ijournal.us/issue_09/ij_issue09_09_MeehanAndHuntsman_01.html

The Pathways Through Algebra Project, founded in 1998 by a group of community college mathematics faculty, has been focusing on improving the student success rates in Elementary Algebra. The following three articles provide some insight into the issues that the Pathways Project is facing. They first appeared in the Pathways Annual Report for 2003/4. The first article by Ken Meehan presents statewide data on student performance in elementary algebra and the impact and effect of some Pathways innovation projects upon success rates. The next article by Hal Huntsman summarizes the findings of a statewide survey of mathematics departments in the community colleges, specifically examining how departments are addressing the delivery of algebra instruction. The final piece, also by Huntsman, discusses the student perspective on learning algebra in the community college setting. For more information about the Pathways project contact Terrie Teegarden at tteegard@sdccd.cc.ca.us.

Table of Contents

Please click on the title or scroll down for access to each article.

[Pathways Report on Student Retention and Success in Algebra in California's Community Colleges](#)

By Kenneth Meehan, Ph.D.

[Pathways State Survey on Teaching Algebra](#)

By Hal Huntsman

[Pathways Focus on Student Perspectives of Learning Algebra](#)

By Hal Huntsman

Pathways Report on Student Retention and Success in Algebra in California's Community Colleges

Kenneth Meehan, Ph.D.

The Pathways through Algebra Project initially focused on Elementary Algebra because of the low successful course completion rates in that course throughout the California Community Colleges. The California Community Colleges Chancellor's Office defines success as the proportion of all students enrolled at the census date who earn grades of A, B, C, or Credit. Retention, or course completion, is defined as the proportion of all students enrolled at the census date who do not withdraw before the end of the class. The original analysis of statewide data from 1998-99 indicated that the average success rate throughout the system was approximately 46%.

A subsequent analysis of statewide data was conducted in the spring of 2004, examining data from the then 108 California community colleges from Spring 2001 through Fall 2003 for both elementary and intermediate algebra. Table 1 presents the retention and success rates for all colleges for each year.

Table 1. Retention and Success in Elementary and Intermediate Algebra

Level	Year	Enrollments	Retention Rate	Success Rate
Elementary	2001	71,275	71.7%	47.5%
	2002	74,907	72.2%	49.7%
	2003	71,676	73.2%	49.2%
	Total	217,858	72.4%	48.8%
Intermediate	2001	60,484	73.1%	50.8%
	2002	63,344	73.2%	51.8%
	2003	62,022	73.7%	51.5%
	Total	185,850	73.3%	51.3%

The data were subsequently disaggregated by age, gender, and race. Tables 2 through 7 present these results.

Table 2. Retention and Success in Elementary Algebra by Gender

Gender	Enrollment	Retention Rate	Success Rate
Female	296,237	73.5 %	51.9 %
Male	191,876	71.5 %	46.4 %
Total	488,113	72.7 %	49.6 %

Table 3. Retention and Success in Elementary Algebra by Age

Age	Enrollment	Retention Rate	Success Rate
19 or younger	186,295	74.4 %	46.3 %
20 - 24	152,736	70.6 %	46.4 %
25 - 29	51,666	72.4 %	54.7 %
30 - 34	32,026	73.2 %	57.7 %
35 - 39	24,054	73.3 %	59.0 %
40 - 49	32,449	73.2 %	59.4 %
50 or older	10,776	69.7 %	55.4 %
Total	490,002	72.7 %	49.7 %

Table 4. Retention and Success in Elementary Algebra by Race

Race	Enrollment	Retention Rate	Success Rate
African-American	48,650	66.3 %	40.2 %
American Indian	5,138	69.5 %	44.4 %
Asian Pacific Islander	55,402	76.1 %	54.8 %
Hispanic	167,906	71.5 %	46.8 %
Other (Non-white)	9,652	72.2 %	47.9 %
White	180,292	74.5 %	53.5 %
Total	467,040	72.7 %	49.6 %

Table 5. Retention and Success in Intermediate Algebra by Gender

Gender	Enrollment	Retention Rate	Success Rate
Female	237,711	75.2 %	54.8 %
Male	177,048	73.2 %	50.1 %
Total	414,759	74.3 %	52.8 %

Table 6. Retention and Success in Intermediate Algebra by Age

Age	Enrollment	Retention Rate	Success Rate
19 or younger	171,469	76.9 %	52.5 %
20 - 24	147,780	71.7 %	49.0 %
25 - 29	40,236	73.0 %	56.0 %
30 - 34	20,823	73.7 %	59.3 %
35 - 39	13,608	74.5 %	61.5 %
40 - 49	17,299	75.2 %	63.3 %
50 or older	5,256	73.7 %	60.5 %
Total	416,471	74.3 %	52.8 %

Table 7. Retention and Success in Intermediate Algebra by Race

Race	Enrollment	Retention Rate	Success Rate
African-American	28,139	68.1 %	42.5 %
American Indian	3,833	70.0 %	47.0 %
Asian Pacific Islander	63,675	77.7 %	57.0 %
Hispanic	123,875	72.0 %	48.6 %
Other (Non-white)	8,859	73.9 %	52.5 %
White	166,058	75.9 %	56.0 %
Total	394,429	74.3 %	52.7 %

The initial analysis of the statewide data revealed significant differences by gender, age, and race and those differences persist through the most recent analyses. Females have significantly higher success rates than males, older students achieve higher success rates than younger students, and whites and Asians outperform African-American, American Indian and Hispanic students. These results echo those of other studies that indicate that the most at-risk population is young, male students of color.

Impact of Pathways Interventions

In a series of pilot studies, the Pathways through Algebra project attempted several interventions to address the lack of success in elementary algebra. The three interventions consisted of (1) a computer assisted course, (2) a mathematics study center, and (3) a mathematics study skills course. The intervention groups were matched with other elementary algebra classes taught in traditional fashion. Table 8 presents the results of the intervention through examination of success rates.

Table 8. Success Rates for Intervention and Control Elementary Algebra Classes

Intervention	Intervention Classes	Control Classes
Computer Assisted	44.1 %	48.6 %
Study Center	60.3 %	39.2 %
Study Skills	66.7 %	53.6 %
Total	58.2 %	40.9 %

The results are mixed but promising. For two of the interventions, the classes participating in the intervention significantly outperformed the control classes, while the computer assisted class had a slightly lower success rate. Refinements to the initial interventions have been made after examining the pilot results.

Dr. Kenneth Meehan is the Director of Institutional Research at Fullerton College and the senior researcher for the Center for Student Success for the Algebra Pathways Project.

Pathways State Survey on Teaching Algebra

Hal Huntsman

During Spring 2004, Pathways asked every community college in California to answer a simple survey about elementary and intermediate algebra. We received 68 responses, 61 of which were from separate institutions (out of the possible 109)¹. The data are rich and this report is a first attempt to draw inferences from it.

Summary

Our survey responses suggest that at least 50% of elementary and intermediate algebra courses are taught by part-time faculty. In addition, the survey shows there are many instructors and institutions developing new and innovative ideas with the goal of improving algebra student success. On the other hand, there are barriers to the success of new initiatives:

- Lack of resources;
- Institutional and, perhaps more significantly, departmental resistance;
- Lack of access to data with which to analyze their approaches.

Half Part-Time Faculty

According to the data, full-time instructors teach less than 50% of the elementary and intermediate algebra classes, state-wide. Thus, if we are going to have a real impact on the success of our algebra students, oft-neglected part-time instructors must receive professional development and access to data at least equivalent to that given full-time faculty.

Of the 61 schools responding, 37 report they are using an alternative approach to teaching algebra. Evergreen College is experimenting with four or five different programs—some aimed at specific demographic groups—but all designed to connect learners with mathematics in more and different ways than traditional lecture courses do. The attempt to create context and increase student motivation to learn mathematics has been taken even further by instructors at the College of San Mateo and Orange Coast College, who have independently developed entirely new curricula. Other alternative approaches include:

- Required tutoring lab hours;
- Computer-assisted instruction combined with traditional lecture;
- Collaborative learning;
- Elementary algebra offered as a two-semester sequence;
- Learning communities with study skills or English courses;
- Graphing calculators in the classroom;
- A "reformed" style curriculum.

The number and range of these approaches should encourage us. A crucial next step is to identify what is working best, for what kinds of students and for what kinds of instructors.

Additional Resources Needed

Respondents report that they need the following things to help them improve student success:

- More tutoring;
- More time to cover topics;

- Better prepared/motivated students;
- More support—time, money, and other resources;
- Smaller classes;
- Training in alternative teaching techniques;
- More and better communication among faculty and between faculty and administrators;

These points coincide with the responses to the survey's question about institutional support for change in the department and classroom, responses that boil down to two issues:

- Institutional talk supporting change is common, but resource allocation to enable that change is less so. Resources include, but are not limited to, resources for tutors, release time for curricular development, classroom allocation, resources for professional conferences, and reduced class size.
- Resistance to (or at least lack of support for) curricular and pedagogical innovation can come both from a college's administration and from within the mathematics department. Often the intradepartmental opposition is the more intractable and results in marginalization of faculty committed to improving instruction.

These data suggest that enabling change requires both resources and dialog. Innovators need to be talking with their colleagues both within their departments and within their institutions as a whole; consensus must be built so that support for good teaching and programs comes in the form of both resources and institutional and departmental validation.

Lack of Clear Data and Accurate Communication

Lack of communication and information within institutions and departments is also the likely explanation for the surveys we received from the six institutions from which we received multiple responses. None of the multiple responses give consistent answers to all questions. There is disagreement even about seemingly straightforward things, such as how many sections of elementary or intermediate algebra are offered at that college. More to the point, faculty at the same institutional seem to be unaware of the new approaches their colleagues in the same department—not to mention in the district, state, and nation—are developing. Access to institutional data is an especially acute issue at smaller colleges without full-time research officers.

Pathways sees this survey and its results as a first pass at the issues and we welcome your comments and questions regarding this write up. You can e-mail Hal Huntsman at shuntsma@ccsf.edu or Terrie Teegarden at tteegard@sdccd.cc.ca.us. We look forward to the continuing dialogue.

If you would like to see a copy of the survey we used and/or would like to see the results in more detail please go to: <http://research.ccsf.edu/Sample/PathWays.asp>.

Hal Huntsman is a Certified Developmental Specialist, the Mathematics Lab Coordinator at City College of San Francisco (CCSF), and the newest member of the Pathways team.

¹ 25 of 61 institutional surveys were completed by a known department chair or division dean. The other 36 were completed by regular classroom instructors.

Pathways Focus on Student Perspectives of Learning Algebra

Hal Huntsman

Student feedback can be an important source of ideas to increase algebra success. With that in mind, Pathways conducted a series of student focus groups: April 22, 2004, at Laney College; June 11, 2004 at Southwestern College; and April 30, 2004, with students from the Delano Campus of Bakersfield College. This last group also presented their thoughts to faculty and administrators at the Spring Pathways conference.

What do students say when asked what teachers should (and shouldn't) do to help them learn algebra? Asking this question and listening carefully to the answers highlighted an important fact for me: too often we as teachers do not listen to our students. Too often the focus in our classrooms is on the teacher, not the student, and as a result, we don't hear the students.

Mathematics Fears and Anxieties

What I heard them say was that even though they are "intimidated by math and math teachers," they want to learn and they want their instructors to connect with them. Students feel that teachers forget how hard it was to learn this material the first time and that too often instructors don't understand how their students feel. One student suggested that she would like her instructors to research the high schools and the community so that they "understand the kinds of schools we come from, how much we know, and what kind of culture we live in." Many students stressed the need for teachers to be patient and not to "show frustration when we're not getting it." They feel as though their teacher doesn't care if they hear a comment like "you should already know this" or if the teacher "explains the problem once and then sends us away." They especially don't want to be embarrassed in front of the class, and even very subtle comments about grades or progress can fall into that category. In addition, teachers should be careful about the use of sarcasm; many students—particularly non-native English speakers—miss the tone of voice and absorb the words, thereby creating bad feelings and barriers to learning.

Making Algebra More Interesting

There were many suggestions about how to make mathematics more interesting, easier to learn and less confusing. Students want teachers to "help us get involved. Small groups, games with competition, helping each other learn—things like that keep us interested, not falling asleep." Along with that, "teachers need to pay attention to whether a class is getting it or not. If the class is understanding, the teacher needs to move on. If they aren't getting it, the teacher should slow down." They recommend using "step-by-step" descriptions of material and avoiding explanations that differ too much from the book. Of equal importance, students would like instructors to "answer questions as soon as you can, [because] when I get put off too long, I forget my question and lose track of why I asked in the first place." Students also say they learn better when homework is assigned, turned in, and graded every day. They like frequent reviews and quizzes, as well as shorter tests more often, and when pushed for specifics recommended testing as often as once a week. Finally, some students complain that "our teacher uses too many hard words and the book does it, too. It's even worse when English isn't your first language."

Instructor Preparation

There was another whole category of comments regarding instructor preparation, a topic that came out spontaneously from the students: "We can tell when teachers aren't ready for class and it feels like they're just wasting our time. We feel disrespected." They want teachers to be prepared for class, ready with

examples and activities. Students even suggested that "teachers should get together and share what kinds of things work," a great idea if we at Pathways ever heard one.

Diverse Student Profile

The students who made these comments are remarkably varied. For example, the Laney group had several students who had repeated Elementary Algebra, sometimes more than once. They were African-American, Asian, Latina/o/a, Caucasian. They struggled and they sought help, but in some cases were turned off by teachers and tutors that "run from you," "act above," and "don't really want to help." When that happens, they stop seeking help from those sources. Despite these obstacles, all these students tried again; they persisted, piecing together whatever help they could get from friend, classmates, and the teachers who show that they care.

On the other hand, the students from Bakersfield College are mostly successful mathematics students, some of whom want to become teachers. Their campus didn't have tutors available until recently, and when they struggle they have learned to rely on each other. As a result, they are a tightly knit group of women and men who help each other translate their texts and notes and teachers into Spanish, help each other with concepts, and share their troubles and successes and ideas. Their parents are proud of them and don't want their children to be field-working "mules" like they are. And despite all this, these students thought of themselves as "average," a feeling my fellow interviewer and I explicitly questioned, in the hope that they might see how very extraordinary they are.

Underlying all the students' comments and stories were the desires to be heard, to be respected, to have teachers that care about them, and to learn—something they think they can do if given the opportunity and encouragement. Hearing these desires is an important first step toward meeting them.

Hal Huntsman is a Certified Developmental Specialist, the Mathematics Lab Coordinator at City College of San Francisco (CCSF), and the newest member of the Pathways team.

Appendix 3

Mathematics Study Skills: Diagnostic Inventory

Rate your achievement of the following statements by rating yourself **3** for **almost always**, **2** for **sometimes**, **1** for **almost never**, **0** if you have never even thought about doing what the statement says.

Selecting a Mathematics Class

1. I schedule my math class at a time when I am mentally sharp.
2. When I register for a math class, I choose the best instructor for me.
3. If I have a choice, I select a math class that meets three or four days a week instead of one or two.
4. I schedule my next math class as soon as possible after I have completed the current course.
5. I am sure that I have signed up for the correct level math course.

During Math Class

6. I come to class on time and even try to be early.
7. I sit as close to front and center of the room as possible.
8. Before class starts I review my notes
9. I never miss class.
10. If I must miss class, I get clear accurate notes and homework assignment and try to work assignment before the next class.
11. I make a conscious effort to focus each class period.
12. My goal for each class is to learn as much as possible.
13. I try to find a way connect new concepts to what I already know.
14. I take good notes in class.
15. I have a method for taking good notes.
16. I ask questions when I don't understand.
17. If I get lost, I identify where I got lost.
18. I attend additional classes if I need to go through it again.

Time and Place for Studying Math

- 19. I study math every day.
- 20. I try to do my math homework immediately after math class.
- 21. I have a specific time to study math.
- 22. I have a specific place with few distractions to study math.
- 23. I do my math homework in the lab where I can get help.
- 24. I am careful to keep up to date with my math homework.
- 25. I study math at least eight to ten hours a week.
- 26. I study in short sessions 45-65 minutes.

Study Strategies for Math Class

- 27. I read my math textbook before I come to class.
- 28. If I have trouble understanding the textbook, I find an alternative text.
- 29. I take notes in math class.
- 30. I am careful to copy all the steps of math problems in my notes.
- 31. I ask questions when I am confused.
- 32. I go to the instructor or lab when I am confused.
- 33. I try to determine exactly when I got confused and exactly what confused me.
- 34. I review my notes and text before beginning homework.
- 35. I work problems until I understand them, not just until I get the right answer for homework.
- 36. I use flash cards for formulas and vocabulary.
- 37. I develop memory techniques to remember math concepts.

Math Tests

- 38. I preview the test before I begin.
- 39. Before I begin taking the test, I make notes on the test of things such as formulas that I might need or forget.
- 40. I begin with the easy questions first.
- 41. I take the full amount of time allotted for the test.
- 42. I carefully check or rework as many problems as possible before I turn in my test.

43. When tests are returned, I keep a log of the types of mistakes I made—concept errors, application errors, careless errors.
44. I keep up to date so that I don't have to cram the night before a test.

Anxiety

45. I believe that I can succeed in math class.
46. I have study partners in my math class.
47. I find out as much as possible about each test.
48. I take practice tests.
49. I know several good relaxation and breathing techniques.
50. I am comfortable asking for help.

Your final score is:

Here's How To Use Your Score

- If your score is **130-150**, give yourself an **A**. You are using the study skills you need in order to be successful in math.
- If your score is **110-129**, give yourself a **B**. You are using good math study skills. Choose a few strategies to work on each day, and you will be well on your way to an A.
- If your score is **85-109**, give yourself a **C**. Your study skills are average. If you want an A, choose one or two strategies in each category to work on until you are using most of the strategies described in the inventory.
- If your score is below **85**, you are probably having a difficult time in math class. Math may not be your trouble! More than likely, your main problem is the study strategies you are using (or not using). Make yourself do as many of the fifty things listed as you can.

Used with permission

Hopper, C. (2007). *Math Study Skills: Diagnostic Inventory*. Practicing College Learning Strategies, 4th edition, Houghton Mifflin

Appendix 4
Statement on Competencies in Mathematics Expected of
Entering College Students

Committee Members

SCOTT FARRAND (Committee Chair), CSU Sacramento

JOAN CORDOVA, Orange Coast College

PHILIP CURTIS, UC Los Angeles

PATRICIA DEAMER, Skyline College

MARGARET DeARMOND, East Bakersfield High School

WALTER DENHAM, California Department of Education

JOSE GUTIERREZ, San Francisco State University

EUNICE KRINSKY, CSU Dominguez Hills

ALFRED MANASTER, UC San Diego

DAVID MORIN, East Los Angeles College

TOM SALLEE, UC Davis

BETH SCHLESINGER, San Diego High School

Statement on Competencies in Mathematics

Introduction

The goal of this Statement on Competencies in Mathematics Expected of Entering College Students is to provide a clear and coherent message about the mathematics that students need to know and to be able to do to be successful in college. While this is written especially for the secondary mathematics teachers, it should be useful for anyone who is concerned about the preparation of California's students for college. This represents an effort to be realistic about the skills, approaches, experiences, and subject matter that make up an appropriate mathematical background for entering college students.

The first section describes some characteristics that identify the student who is properly prepared for college courses that are quantitative in their approach. The second section describes the background in technology, such as calculators, that college students should have. The third section describes the subject matter that is an essential part of the background for all entering college students, as well as describing what is the essential background for students intending quantitative majors. Among the descriptions of subject matter there are sample problems. These are intended to clarify the descriptions of subject matter and to be representative of the appropriate level of understanding. The sample problems do not cover all of the mathematical topics identified.

No section of this Statement should be ignored. Students need the approaches, attitudes, and perspectives on mathematics described in the first section. Students need the experiences with technology described in the second section. And students need extensive skills and knowledge in the subject matter areas described in the third section. Inadequate attention to any of these components is apt to disadvantage the student in ways that impose a serious impediment to success in college. Nothing less than a balance among these components is acceptable for California's students.

The discussion in this document of the mathematical competencies expected of entering college students is predicated on the following basic recommendation:

For proper preparation for baccalaureate level course work, all students should be enrolled in a mathematics course in every semester of high school. It is particularly important that students take mathematics courses in their senior year of high school, even if they have completed three years of college preparatory mathematics by the end of their junior year. Experience has shown that students who take a hiatus from the study of mathematics in high school are very often unprepared for courses of a quantitative nature in college and are unable to continue in these courses without remediation in mathematics.

Section 1

Approaches to Mathematics

This section enumerates characteristics of entering freshmen college students who have the mathematical maturity to be successful in a first college mathematics course, and in other college courses that are quantitative in their approach. These characteristics are described primarily in terms of how students approach mathematical problems. The second part of this section provides suggestions to secondary teachers of ways to present mathematics that will help their students to develop these characteristics.

Part 1

Dispositions of well-prepared students toward mathematics

Crucial to their success in college is the way in which students encounter the challenges of new problems and new ideas. From their high school mathematics courses students should have gained certain approaches, attitudes, and perspectives:

- A view that mathematics makes sense—students should perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized and applied.
- An ease in using their mathematical knowledge to solve unfamiliar problems in both concrete and abstract situations - students should be able to find patterns, make conjectures, and test those conjectures; they should recognize that abstraction and generalization are important sources of the power of mathematics; they should understand that mathematical structures are useful as representations of phenomena in the physical world; they should consistently verify that their solutions to problems are reasonable.
- A willingness to work on mathematical problems requiring time and thought, problems that aren't solved by merely mimicking examples that have already been seen—students should have enough genuine success in solving such problems to be confident, and thus to be tenacious, in their approach to new ones.
- A readiness to discuss the mathematical ideas involved in a problem with other students and to write clearly and coherently about mathematical topics—students should be able to communicate their understanding of mathematics with peers and teachers using both formal and natural languages correctly and effectively.
- An acceptance of responsibility for their own learning - students should realize that their minds are their most important mathematical resource, and that teachers and other students can help them to learn but can't learn for them.
- The understanding that assertions require justification based on persuasive arguments, and an ability to supply appropriate justifications—students should habitually ask "Why?" and should have a familiarity with reasoning at a variety of levels of formality, ranging from concrete examples through informal arguments using words and pictures to precise structured presentations of convincing arguments.
- An openness to the use of appropriate technology, such as graphing calculators and computers, in solving mathematical problems and the attendant awareness of the limitations of this technology—students should be able to make effective use of the technology, which includes the ability to determine when technology will be useful and when it will not be useful.

- A perception of mathematics as a unified field of study-students should see interconnections among various areas of mathematics, which are often perceived as distinct.

Part 2

Aspects of Mathematics Instruction to Foster Student Understanding and Success

There is no best approach to teaching, not even an approach that is effective for all students, or for all instructors. One criterion that should be used in evaluating approaches to teaching mathematics is the extent to which they lead to the development in the student of the dispositions, concepts, and skills that are crucial to success in college. It should be remembered that in the mathematics classroom, time spent focused on mathematics is crucial. The activities and behaviors that can accompany the learning of mathematics must not become goals in themselves-understanding of mathematics is always the goal.

While much has been written recently about approaches to teaching mathematics, as it relates to the preparation of students for success in college, there are a few aspects of mathematics instruction that merit emphasis here.

Modeling Mathematical Thinking

Students are more likely to become intellectually venturesome if it is not only expected of them, but if their classroom is one in which they see others, especially their teacher, thinking in their presence. It is valuable for students to learn with a teacher and others who get excited about mathematics, who work as a team, who experiment and form conjectures. They should learn by example that it is appropriate behavior for people engaged in mathematical exploration to follow uncertain leads, not always to be sure of the path to a solution, and to take risks. Students should understand that learning mathematics is fundamentally about inquiry and personal involvement.

Solving Problems

Problem solving is the essence of mathematics. Problem solving is not a collection of specific techniques to be learned; it cannot be reduced to a set of procedures. Problem solving is taught by giving students appropriate experience in solving unfamiliar problems, by then engaging them in a discussion of their various attempts at solutions, and by reflecting on these processes. Students entering college should have had successful experiences solving a wide variety of mathematical problems. The goal is the development of open, inquiring, and demanding minds. Experience in solving problems gives students the confidence and skills to approach new situations creatively, by modifying, adapting, and combining their mathematical tools; it gives students the determination to refuse to accept an answer until they can explain it.

Developing Analytic Ability and Logic

A student who can analyze and reason well is a more independent and resilient student. The instructional emphasis at all levels should be on a thorough understanding of the subject matter and the development of logical reasoning. Students should be asked "Why?" frequently enough that they anticipate the question, and ask it of themselves. They should be expected to construct compelling arguments to explain why, and to understand a proof comprising a significant sequence of implications. They should be expected to question and to explore why one statement follows from another. Their understandings should be challenged with questions that cause them to further examine their reasoning. Their experience with mathematical proof should not be limited to the format of a two-column proof; rather, they should see, understand,

and construct proofs in various formats throughout their course work. A classroom full of discourse and interaction that focuses on reasoning is a classroom in which analytic ability and logic are being developed.

Experiencing Mathematics in Depth

Students who have seen a lot but can do little are likely to find difficulty in college. While there is much that is valuable to know in the breadth of mathematics, a shallow but broad mathematical experience does not develop the sort of mathematical sophistication that is most valuable to students in college. Emphasis on coverage of too many topics can trivialize the mathematics that awaits the students, turn the study of mathematics into the memorization of discrete facts and skills, and divest students of their curiosity. By delving deeply into well-chosen areas of mathematics, students develop not just the self-confidence but the ability to understand other mathematics more readily, and independently.

Appreciating the Beauty and Fascination of Mathematics

Students who spend years studying mathematics yet never develop an appreciation of its beauty are cheated of an opportunity to become fascinated by ideas that have engaged individuals and cultures for thousands of years. While applications of mathematics are valuable for motivating students, and as paradigms for their mathematics, an appreciation for the inherent beauty of mathematics should also be nurtured, as mathematics is valuable for more than its utility. Opportunities to enjoy mathematics can make the student more eager to search for patterns, for connections, for answers. This can lead to a deeper mathematical understanding, which also enables the student to use mathematics in a greater variety of applications. An appreciation for the aesthetics of mathematics should permeate the curriculum and should motivate the selection of some topics.

Building Confidence

For each student, successful mathematical experiences are self-perpetuating. It is critical that student confidence be built upon genuine successes—false praise usually has the opposite effect. Genuine success can be built in mathematical inquiry and exploration. Students should find support and reward for being inquisitive, for experimenting, for taking risks, and for being persistent in finding solutions they fully understand. An environment in which this happens is more likely to be an environment in which students generate confidence in their mathematical ability.

Communicating

While solutions to problems are important, so are the processes that lead to the solutions and the reasoning behind the solutions. Students should be able to communicate all of this, but this ability is not quickly developed. Students need extensive experiences in oral and written communication regarding mathematics, and they need constructive, detailed feedback in order to develop these skills. Mathematics is, among other things, a language, and students should be comfortable using the language of mathematics. The goal is not for students to memorize an extensive mathematical vocabulary, but rather for students to develop ease in carefully and precisely discussing the mathematics they are learning. Memorizing terms that students don't use does not contribute to their mathematical understanding. However, using appropriate terminology so as to be precise in communicating mathematical meaning is part and parcel of mathematical reasoning.

Becoming Fluent in Mathematics

To be mathematically capable, students must have a facility with the basic techniques of mathematics. There are necessary skills and knowledge that students must routinely exercise without hesitation. Mathematics is the language of the sciences, and thus fluency in this language is a basic skill. College mathematics classes require that students bring with them ease with the standard skills of mathematics that allows them to focus on the ideas and not become lost in the details. However, this level of internalization of mathematical skills should not be mistaken for the only objective of secondary mathematics education. Student understanding of mathematics is the goal. In developing a skill, students first must develop an understanding. Then as they use the skill in different contexts, they gradually wean themselves from thinking about it deeply each time, until its application becomes routine. But their understanding of the mathematics is the map they use whenever they become disoriented in this process. The process of applying skills in varying and increasingly complex applications is one of the ways that students not only sharpen their skills, but also reinforce and strengthen their understanding. Thus, in the best of mathematical environments, there is no dichotomy between gaining skills and gaining understanding. A curriculum that is based on depth and problem solving can be quite effective in this regard provided that it focuses on appropriate areas of mathematics.

Section 2

Technology

The pace at which advances are made in technology, and the surprising ways in which mathematics pedagogy and curriculum change in response to those advances, make it impossible to anticipate what technological experiences and skills students will need for success in college in the coming years. Also, the diversity of responses to technology among the college mathematics courses in California further impede the development of a clear statement on the appropriate technological background for entering college students. But the general directions are discernible. The past has shown us that scientific calculators make many problems accessible to students that previously were not because of excessive computation. More recently, we've seen that students can use the graphing capabilities of calculators to deepen their understanding of functions. And now the advent of hand-held calculators that perform symbolic algebra computations will certainly have a major impact on the instruction in algebra and more advanced courses.

From all of this, it is clear that entering college students must have availed themselves of opportunities presented by technology. The kind of graphing calculator or computer software preferred at different institutions, by different instructors, in different courses, at different times will of course vary. So, student experiences should not focus on the intricacies of a specific device so much as on the use of technology as a valuable tool in many aspects of their mathematics courses. Entering college students should have considerable experience in the following areas:

- Deciding when to use technology. Students should be able to determine what algebraic or geometric manipulations are necessary to make best use of the calculator. At the same time, they should also be able to determine for themselves when using a calculator, for example, might be advantageous in solving a problem.

- Dealing with data. Students should work on problems posed around real data and involving significant calculations. With repeated applications requiring computation, they can gain skill in estimation, approximation, and the ability to tell if a proposed solution is reasonable. Students should find opportunities to work with data in algebra, geometry, and statistics.
- Checking their Calculations. Whenever possible students should use a calculator with a multi-line screen so that they are able to review the input to the calculator and to determine whether any errors have been made.
- Representing problems geometrically. Students should be able to use graphing calculators as a tool to represent functions and to develop a deeper understanding of domain, range, arithmetic operations on functions, inverse functions, and function composition.
- Experimenting, making conjectures, and finding counterexamples. Students should be comfortable using technology to check their guesses, to formulate revised guesses, and to make conjectures based on these results. They should also challenge conjectures, and find counterexamples. Where possible, they should use tools such as geometric graphing utilities to make and test geometric conjectures and to provide counterexamples.

Section 3

Subject Matter

Decisions about the subject matter for secondary mathematics courses are often difficult, and are too-easily based on tradition and partial information about the expectations of the colleges. What follows is a description of mathematical areas of focus that are (1) essential for all entering college students; (2) desirable for all entering college students; (3) essential for college students to be adequately prepared for quantitative majors; and (4) desirable for college students who intend quantitative majors. This description of content will in many cases necessitate adjustments in a high school mathematics curriculum, generally in the direction of deeper study in the more important areas, at the expense of some breadth of coverage.

Sample problems have been included to indicate the appropriate level of understanding for some areas. The problems included do not cover all of the mathematical topics described, and many involve topics from several areas. Entering college students working independently should be able to solve problems like these in a short time-less than half an hour for each problem included. Students must also be able to solve more complex problems requiring significantly more time.

Part 1

Essential areas of focus for all entering college students

What follows is a summary of the mathematical subjects that are an essential part of the knowledge base and skill base for all students who enter higher education. Students are best served by deep mathematical experiences in these areas. This is intended as a brief compilation of the truly essential topics, as opposed to topics to which students should have been introduced but need not have mastered. The skills and content knowledge that are prerequisite to high school mathematics courses are of course still necessary for success in college, although they are not explicitly mentioned here. Relative to traditional practice, topics and perspectives are described here as appropriate for increased emphasis (which does not mean paramount importance) and for decreased emphasis (which does not mean elimination).

- Variables, Equations, and Algebraic Expressions:** Algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols; solutions of linear equations and inequalities; absolute value; powers and roots; solutions of quadratic equations; solving two linear equations in two unknowns including the graphical interpretation of a simultaneous solution. Increased emphasis should be placed on algebra both as a language for describing mathematical relationships and as a means for solving problems, while decreased emphasis should be placed on interpreting algebra as merely a set of rules for manipulating symbols.

The braking distance of a car (how far it travels after the brakes are applied until it comes to a stop) is proportional to the square of its speed.

Write a formula expressing this relationship and explain the meaning of each term in the formula.

If a car traveling 50 miles per hour has a braking distance of 105 feet, then what would its braking distance be if it were traveling 60 miles per hour?

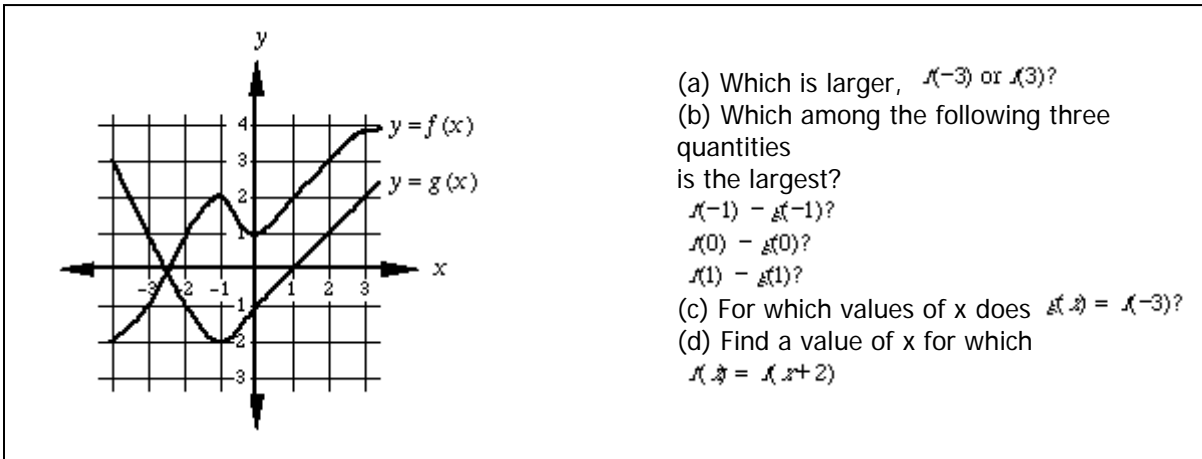
Solve for x and give a reason for each step:

$$\frac{2}{3x+1} + 2 = \frac{2}{3}$$

United States citizens living in Switzerland must pay taxes on their income to both the United States and to Switzerland. The United States tax is 28% of their taxable income after deducting the tax paid to Switzerland. The tax paid to Switzerland is 42% of their taxable income after deducting the tax paid to the United States. If a United States citizen living in Switzerland has a taxable income of \$75,000, how much tax must that citizen pay to each of the two countries? Find these values in as many different ways as you can; try to find ways both using and not using

- Families of Functions and Their Graphs:** Applications; linear functions; quadratic and power functions; exponential functions; roots; operations on functions and the corresponding effects on their graphs; interpretation of graphs; function notation; functions in context, as models for data. Increased emphasis should be placed on various representations of functions-using graphs, tables, variables, words -and on the interplay among the graphical and other representations, while decreased emphasis should be placed on repeated manipulations of algebraic expressions.

Car dealers use the "rule of thumb" that a car loses about 30% of its value each year. Suppose that you bought a new car in December 1995 for \$20,000. According to this "rule of thumb," what would the car be worth in December 1996? In December 1997? In December 2005? Develop a general formula for the value of the car t years after purchase.



- Find a quadratic function of x that has zeroes at $x = -1$ and $x = 2$.
 Find a cubic function of x that has zeroes at $x = -1$ and $x = 2$ and nowhere else.

Geometric Concepts: Distances, areas, and volumes, and their relationship with dimension; angle measurement; similarity; congruence; lines, triangles, circles, and their properties; symmetry; Pythagorean Theorem; coordinate geometry in the plane, including distance between points, midpoint, equation of a circle; introduction to coordinate geometry in three dimensions; right angle trigonometry. Increased emphasis should be placed on developing an understanding of geometric concepts sufficient to solve unfamiliar problems and an understanding of the need for compelling geometric arguments, while decreased emphasis should be placed on memorization of terminology and formulas.

A contemporary philosopher wrote that in 50 days the earth traveled approximately 40 million miles along its orbit and that the distance between the positions of the earth at the beginning and the end of the 50 days was approximately 40 million miles. Discuss any errors you can find in these conclusions or explain why they seem to be correct. You may approximate the earth's orbit by a circle with radius 93 million miles.

$ABCD$ is a square and the midpoints of the sides are $E, F, G,$ and H . $AB = 10$ in. Use at least two different methods to find the area of parallelogram $AFCH$.

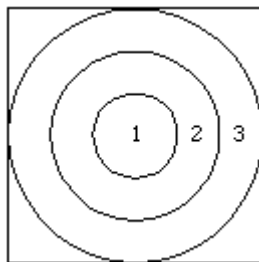
Two trees are similar in shape, but one is three times as tall as the other. If the smaller tree weighs two tons, how much would you expect the larger tree to weigh? Suppose that the bark from these trees is broken up and placed into bags for landscaping uses. If the bark from these trees is the same thickness on the smaller tree as the larger tree, and if the larger tree yields 540 bags of bark, how many bags would you expect to get from the smaller tree?

An 82 in. by 11 in. sheet of paper can be rolled lengthwise to make a cylinder, or it can be rolled widthwise to make a different cylinder. Without computing the volumes of the two cylinders, predict which will have the greater volume, and explain why you expect that. Find the volumes of the two cylinders to see if your prediction was correct. If the cylinders are to be covered top and bottom with additional paper, which way of rolling the cylinder will give the greater total surface area?

- **Probability:** Counting (permutations and combinations, multiplication principle); sample spaces; expected value; conditional probability; area representations of probability. Increased emphasis should be placed on a conceptual understanding of discrete probability, while decreased emphasis should be placed on aspects of probability that involve memorization and rote application of formulas.

If you take one jellybean from a large bin containing 10 lbs. of jellybeans, the chance that it is cherry flavored is 20%. How many more pounds of cherry jelly beans would have to be mixed into the bin to make the chance of getting a cherry one 25%?

A point is randomly illuminated on a computer game screen that looks like the figure shown below.



The radius of the inner circle is 3 inches; the radius of the middle circle is 6 inches; the radius of the outer circle is 9 inches.

What is the probability that the illuminated point is in region 1?

What is the probability that the illuminated point is in region I if you know that it isn't in region 2?

A fundraising group sells 1000 raffle tickets at \$5 each. The first prize is an \$1,800 computer. Second prize is a \$500 camera and the third prize is \$300 cash. What is the expected value of a raffle ticket?

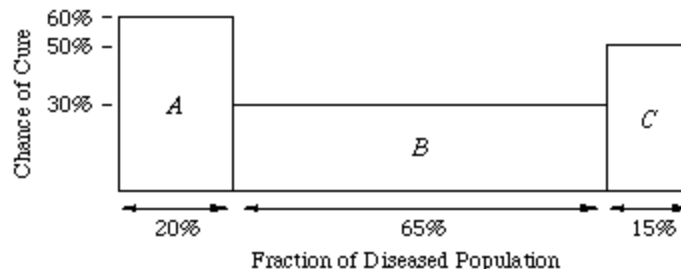
Five friends line up at a movie theater. What is the probability that Mary and Mercedes are standing next to each other?

- Data Analysis and Statistics:** Presentation and analysis of data; mean, median and standard deviation; representative samples; using lines to fit data and make predictions. Increased emphasis should be placed on organizing and describing data and making predictions based on the data, with common sense as a guide, while decreased emphasis should be placed on aspects of statistics that are learned as algorithms without an understanding of the underlying ideas.

The table at the right shows the population of the USA in each of the last five censuses. Make a scatter plot of this data and draw a line on your scatter plot that fits this data well. Find an equation for your line, and use this equation to predict what the population might be in the year 2000. Plot that predicted point on your graph and see if it seems reasonable. What is the slope of your line? Write a sentence that describes to someone who might not know about graphs and lines what the meaning of the slope is in terms involving the USA population.

Year	USA Population (millions)
1950	152.3
1960	180.7
1970	205.1
1980	227.7
1990	249.9

The results of a study of the effectiveness of a certain treatment for a blood disease are summarized in the chart shown below. The blood disease has three types, A, B, and C. The cure rate for each of the types is shown vertically on the chart. The percentage of diseased persons with each type of the disease is shown horizontally on the same chart.



Fraction of Diseased Population

Find the mean and standard deviation of the following seven numbers:

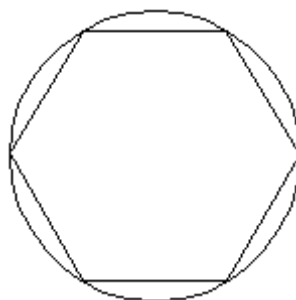
4 12 5 6 8 5 9

- Cha
- Make up another list of seven numbers with the same mean and a smaller standard deviation.
 - Make up another list of seven numbers with the same mean and a larger standard deviation.

- *Argumentation and Proof*: Mathematical implication; hypotheses and conclusions; direct and indirect reasoning; inductive and deductive reasoning. Increased emphasis should be placed on constructing and recognizing valid mathematical arguments, while decreased emphasis should be placed on mathematical proofs as formal exercises.

Select any odd number, then square it, and then subtract one. Must the result always be even? Write a convincing argument.

Use the perimeter of a regular hexagon inscribed in a circle to explain why $\pi > 3$.



Does the origin lie inside of, outside of, or on the geometric figure whose equation is $x^2 + y^2 - 10x + 10y - 1 = 0$? Explain your reasoning.

Part 2

Desirable areas of focus for all entering college students

What follows is a brief summary of some of the mathematical subjects that are a **desirable** part of the mathematical experiences for all students who enter higher education. No curriculum would include study in all of these areas, as that would certainly be at the expense of opportunities for deep explorations in selected areas. But these areas provide excellent contexts for the approaches to teaching suggested in Section I, and any successful high school mathematics program will include some of these topics. The emphasis here is on enrichment and on opportunities for student inquiry.

- *Discrete Mathematics*: Graph theory; coding theory; voting systems; game theory; decision theory.
- *Sequences and Series*: Geometric and arithmetic sequences and series; the Fibonacci sequence; recursion relations.
- *Geometry*: Transformational geometry, including rotations, reflections, translations, and dilations; tessellations; solid geometry; three-dimensional coordinate geometry, including lines and planes.
- *Number Theory*: Prime numbers; prime factorization; rational and irrational numbers; triangular numbers; Pascal's triangle; Pythagorean triples.

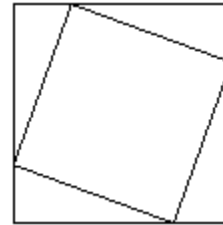
Part 3

Essential areas of focus for students in quantitative majors

What follows is a brief summary of the mathematical subjects that are an **essential** part of the knowledge base and skill base for students to be adequately prepared for quantitative majors. Students are best served by deep mathematical experiences in these areas. The skills and content knowledge listed above as essential for all students entering college are of course also essential for these students-moreover, students in quantitative majors must have a deeper understanding of and a greater facility with those areas.

- *Variables, Equations, and Algebraic Expressions:* Solutions to systems of equations, and their geometrical interpretation; solutions to quadratic equations, both algebraic and graphical; the correspondence between roots and factors of polynomials; the binomial theorem.

In the figure shown to the right, the area between the two squares is 11 square inches. The sum of the perimeters of the two squares is 44 inches. Find the length of a side of the larger square.



Determine the middle term in the binomial expansion of $\left(x - \frac{2}{x}\right)^{10}$

- *Functions:* Logarithmic functions, their graphs, and applications; trigonometric functions of real variables, their graphs, properties including periodicity, and applications; basic trigonometric identities; operations on functions, including addition, subtraction, multiplication, reciprocals, division, composition, and iteration; inverse functions and their graphs; domain and range.

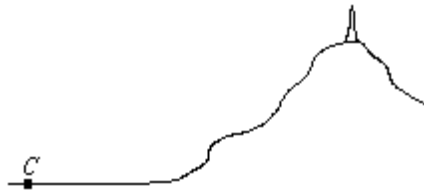
Which of the following functions are their own inverses? Use at least two different methods to answer this, and explain your methods.

$$f(x) = \frac{2}{x}$$
$$g(x) = x^3 + 4$$
$$h(x) = \frac{2 + \ln(x)}{2 - \ln(x)}$$
$$k(x) = \sqrt{\frac{x^2 + 1}{x^2 - 1}}$$

Scientists have observed that living matter contains, in addition to Carbon, C 12, a fixed percentage of a radioactive isotope of Carbon, C14. When the living material dies, the amount of C12 present remains constant, but the amount of C14 decreases exponentially with a half life of 5,550 years. In 1965, the charcoal from cooking pits found at a site in Newfoundland used by Vikings was analyzed and the percentage of C14 remaining was found to be 88.6%. What was the approximate date of this Viking settlement?

Find all quadratic functions of x that have zeroes at $x = -1$ and $x = 2$.
 Find all cubic functions of x that have zeroes at $x = -1$ and $x = 2$ and nowhere else.

A cellular phone system relay tower is located atop a hill. You have a transit and a calculator. You are standing at point C. Assume that you have a clear view of the base of the tower from point C, that C is at sea level, and that the top of the hill is 2000 ft. above sea level.



Describe a method that you could use for determining the height of the relay tower, without going to the top of the hill.

Next choose some values for the unknown measurements that you need in order to find a numerical value for the height of the tower, and find the height of the tower.

- *Geometric Concepts:* Two- and three-dimensional coordinate geometry; locus problems; polar coordinates; vectors; parametric representations of curves.

Find any points of intersection (first in polar coordinates and then in rectangular coordinates) of the graphs of $r = 1 + \sin \theta$ and the circle of radius $\frac{3}{2}$ centered about the origin. Verify your solutions by graphing the curves.

Find any points of intersection (first in polar coordinates and then in rectangular coordinates) of the graphs of $r = 1 + \sin \theta$ and the line with slope 1 that passes through the origin. Verify your solutions by graphing the curves.

Marcus is in his back yard, and has left his stereo and a telephone 24 feet apart. He can't move the stereo or the phone, but he knows from experience that in order to hear the telephone ring, he must be located so that the stereo is at least twice as far from him as the phone. Draw a diagram with a coordinate system chosen, and use this to find out where Marcus can be in order to hear the phone when it rings.

Cha

- A box is twice as high as it is wide and three times as long as it is wide. It just fits into a sphere of radius 3 feet. What is the width of the box?

Select any odd number, then square it, and then subtract one. Must the result always be divisible by 4? Must the result always be divisible by 8? Must the result always be divisible by 16? Write convincing arguments or give counterexamples.

The midpoints of a quadrilateral are connected to form a new quadrilateral. Prove that the new quadrilateral must be a parallelogram.
In case the first quadrilateral is a rectangle, what special kind of parallelogram must the new quadrilateral be? Explain why your answer is correct for any rectangle.

Part 4

Desirable areas of focus for students in quantitative majors

What follows is a brief summary of some of the mathematical subjects that are a **desirable** part of the mathematical experiences for students who enter higher education with the possibility of pursuing quantitative majors. No curriculum would include study in all of these areas, as that would certainly be at the expense of opportunities for deep explorations in selected areas. But these areas each provide excellent contexts for the approaches to teaching suggested in Section 1. The emphasis here is on enrichment and on opportunities for student inquiry.

- *Vectors and Matrices*: Vectors in the plane; complex numbers and their arithmetic; vectors in space; dot and cross product, matrix operations and applications.
- *Probability and Statistics*: Continuous distributions; binomial distributions; fitting data with curves; regression; correlation; sampling.
- *Conic Sections*: Representations as plane sections of a cone; focus-directrix properties; reflective properties.
- *Non-Euclidean Geometry*: History of the attempts to prove Euclid's parallel postulate; equivalent forms of the parallel postulate; models in a circle or sphere; seven-point geometry.
- *Calculus* *

* Students should take calculus only if they have demonstrated a mastery of algebra, geometry, trigonometry, and coordinate geometry. Their calculus course should be treated as a college level course and should prepare them to take one of the College Board's Advanced Placement Examinations. A joint statement from the Mathematical Association of America and the National Council of Teachers of Mathematics concerning calculus in secondary schools is included as Appendix B.

Comments on Implementation

Students who are ready to succeed in college will have become prepared throughout their primary and secondary education, not just in their *college preparatory* high school classes. Concept and skill development in the high school curriculum should be a deliberately coordinated extension of the elementary and middle school curriculum. This will require some changes, and some flexibility, in the planning and delivery of curriculum, especially in the first three years of college preparatory mathematics. For example, student understanding of probability and data analysis will be based on experiences that began when they began school, where they became accustomed to performing experiments, collecting data, and presenting the data. This is a more substantial and more intuitive understanding of probability and data analysis than one based solely on an axiomatic development of probability functions on a sample space, for example. It must be noted that inclusion of more study of data analysis in the first three years of the college preparatory curriculum, although an extension of the K-8 curriculum, will be at the expense of some other topics. The general direction, away from a broad but shallow coverage of algebra and geometry topics, should allow opportunities for this.

Appendix A

What follows is a collection of skills that students must routinely exercise without hesitation in order to be prepared for college work. These are intended as indicators—students who have difficulty with many of these skills are significantly disadvantaged and are apt to require remediation in order to succeed in college courses. This list is not exhaustive of the basic skills. This is also not a list of skills that are sufficient to ensure success in college mathematical endeavors.

The absence of errors in student work is not the litmus test for mathematical preparation. Many capable students will make occasional errors in performing the skills listed below, but they should be in the habit of checking their work and thus readily recognize these mistakes, and should easily access their understanding of the mathematics in order to correct them.

1. Perform arithmetic with signed numbers, including fractions and percentages.
2. Combine like terms in algebraic expressions.
3. Use the distributive law for monomials and binomials.
4. Factor monomials out of algebraic expressions.
5. Solve linear equations of one variable.
6. Solve quadratic equations of one variable.
7. Apply laws of exponents.
8. Plot points that are on the graph of a function.
9. Given the measures of two angles in a triangle, find the measure of the third.
10. Find areas of a right triangles.
11. Find and use ratios from similar triangles.
12. Given the lengths of two sides of a right triangle, find the length of the third side.

Appendix B

Calculus in the Secondary School

To: Secondary School Mathematics Teachers

From: The Mathematical Association of America

The National Council of Teachers of Mathematics

Date: September 1986

Re: Calculus in the Secondary School

Dear Colleagues:

A single variable calculus course is now well established in the 12th grade at many secondary schools, and the number of students enrolling is increasing substantially each year. In this letter we would like to discuss two problems that have emerged.

The first problem concerns the relationship between the calculus course offered in high school and the succeeding calculus courses in college. *The Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) recommend that the calculus course offered in the 12th grade should be treated as a college-level course.* The expectation should be that a substantial majority of the students taking the course will master the material and will not then repeat the subject upon entrance to college. Too many students now view their 12th grade calculus course as an introduction to calculus with the expectation of repeating the material in college. This causes an undesirable attitude on the part of the student both in secondary school and in college. In secondary school all too often a student may feel "I don't have to master this material now, because I can repeat it later;" and in college, "I don't have to study this subject too seriously, because I have already seen most of the ideas." Such students typically have considerable difficulty later on as they proceed further into the subject matter.

MAA and NCTM recommend that all students taking calculus in secondary school who are performing satisfactorily in the course should expect to place out of the comparable college calculus course.

Therefore, to verify appropriate placement upon entrance to college, students should either take one of the Advanced Placement (AP) Calculus Examinations of the College Board, or take a locally-administered college placement examination in calculus. Satisfactory performance on an AP examination carries with it college credit at most universities.

A second problem concerns preparation for the calculus course. *MAA and NCTM recommend that students who enroll in a calculus course in secondary school should have demonstrated mastery of algebra, geometry, trigonometry, and coordinate geometry.* This means that students should have at least four full years of mathematical preparation beginning with the first course in algebra. The advanced topics in algebra, trigonometry, analytic geometry, complex numbers, and elementary functions studied in depth during the fourth year of preparation are critically important for students' later courses in mathematics.

It is important to note that at present many well-prepared students take calculus in the 12th grade, place out of the comparable course in college, and do well in succeeding college courses. Currently the two most common methods for preparing students for a college-level calculus course in the 12th grade are to begin the first algebra course in the 8th grade or to require students to take second year algebra and geometry concurrently. Students beginning with algebra in the 9th grade who take only one mathematics course each year in secondary school should not expect to take calculus in the 12th grade. Instead, they should use the 12th grade to prepare themselves fully for calculus as freshman in college.

We offer these recommendations in an attempt to strengthen the calculus program in secondary schools. They are not meant to discourage the teaching of college-level calculus in the 12th grade to strongly prepared students.

Appendix 5

Mathematics Diagnostic Testing Project (MDTP) Scoring Rubrics

The Mathematics Diagnostic Testing Project (MDTP), a joint project of the University of California and the California State University Systems, produces written response items as well as multiple-choice diagnostic and placement tests for middle and high schools, community colleges, and the two systems. Over the last decade, they have produced a general rubric for written response items and implementations of the rubric for specific written response items. Included below are the "General Scoring Rubric for Written Response Items" along with the teacher's version of a pre-algebra level written response item called "Island" including its Essence statement and rubric.

General Scoring Rubric for Written Response Items

Category	Score	Description
No Response	0	Either the work is not attempted (i.e., the paper is blank), or the work is incorrect, irrelevant, or off task. The response may minimally interpret or re-state the problem, but does not go beyond that.
Minimal	1	The response demonstrates only a minimal understanding of the problem posed and a reasonable approach is not suggested. Although there may or may not be some correct mathematical work, the response is incomplete, contains major mathematical errors, or reveals serious flaws in reasoning. Requested examples may be absent or irrelevant.
Partial	2	The response contains evidence of a conceptual understanding of the problem in that a reasonable approach is indicated. However, on the whole, the response is not well developed. Although there may be serious mathematical errors or flaws in reasoning, the response does contain some correct mathematics. Requested examples provided may fail to illustrate the desired conclusions.
Satisfactory	3	The response demonstrates a clear understanding of the problem and provides an acceptable approach. The response also is generally well developed and presented, but contains omissions or minor errors in mathematics. Requested examples provided may not completely illustrate the desired conclusions.
Excellent	4	The response demonstrates a complete understanding of the problem, is correct, and the methods of solution are appropriate and fully developed. The response is logically sound, clearly written, and does not contain any significant errors. Requested examples are well chosen and illustrate the desired conclusions.

EXPLANATORY NOTES

- (1) Rubrics for specific items should always be used with this general rubric and the following notes about specific rubrics.
- (2) The following excerpt from MDTP Guidelines for The Preparation of Written Response Mathematics Questions provides a context for this general rubric. The statement of the question should be explicit and clear. The extent to which students are to discuss their reasoning and results should be explicit. The extent to which students are to provide examples, counterexamples, or generalizations should also be clearly stated.
- (3) Although the categories in the General Scoring Rubric are meant to indicate different levels of understanding and accomplishment, teachers should expect that some student responses may be on the boundary between two categories and may be scored differently by different teachers.
- (4) Teachers may wish to designate some outstanding responses in the Excellent category as exemplars.

NOTES EXPLAINING HOW TO USE SPECIFIC ITEM RUBRICS

Scoring of written responses is to be based upon both the correctness of the mathematics and the clarity of the presentation. In scoring, do NOT “mind read” the presenter; instead only grade the presentation. Grade each response on the actual mathematics written and on the quality of the presentation of that mathematics. Unexecuted recipes or prescriptions should receive minimal credit. The specific scoring rubric for an item outlines the mathematical development necessary for the given scores. In addition to the formal mathematics, it is essential that students “show their work” and clearly present their methodology. The evaluation of each response should be based in part upon its organization, completeness, and clarity. A score of 1 or 2 may in some cases be based simply upon the mathematics called for in the rubric. Scores of 3 and 4 require effective presentation as well as appropriate mathematics. The mathematics called for in specific rubrics is necessary, but not sufficient, for these scores.

Island Essence Statement
Algebra Readiness: Exponents and Square Roots; Scientific Notation

The task is to use population growth information to determine the population of an island at two specific times and to estimate the years in which the population will reach specific levels, one of which is the year when the needs of the population exceed the living space. Both estimates must be supported by explanations. To fully accomplish the task, student work must be correct and clearly presented.

- To find the population in the years 2020 and 2050, student work will show that the population at those times will be 800 people and 1,600 people respectively.
- To find the year when there will be 6,000 people on the island, student work will show that this will occur between the interval 2080 and 2110—but much closer to 2110. (Some student work may show beginning conceptualization of linear interpolation to reach this estimate.)¹
- To find the year when the needs of the population exceed the available living space, student work will demonstrate an understanding that the island can support only 10,000 inhabitants and show that the needs of the population will exceed island space between the years 2110 and 2140 but closer to 2140.²

POSSIBLE EXTENSIONS FOR CLASS ACTIVITY:

Discuss the assumptions about population growth that are built into the above problem. What are some of the factors that could alter this rate of growth and how could it be altered?

Discuss and explore exponential growth, possibly graphing some examples of this function. (E.g., let living space = 1.6 square miles, let doubling time = 50 years, etc.)

¹ Calculation using an exponential growth model puts the time at the year 2107.

² Calculation using an exponential growth model puts the time at the year 2129.

Student Name _____

Class _____



Score

For this written response activity, your work should clearly show how you solved each part.
Label any figures you draw.

A large inhabited island has a total land area of 20,000 square miles. Each person on the island requires an average of 2 square miles of “living space” for housing, food production, and other activities. The population doubles every 30 years. In the year 1990 the population of the island was 400 people.

A. What will the population be in the year 2020? What will the population be in the year 2050?

B. Estimate the year when there will be 6,000 people on the island. Show the work that leads to your estimate.

C. Estimate the year when the needs of the population will exceed the available living space. Show your work and explain how you arrived at your estimate.

For this written response activity, your work should clearly show how you solved each part. Label any figures you draw.

A large inhabited island has a total land area of 20,000 square miles. Each person on the island requires an average of 2 square miles of “living space” for housing, food production, and other activities. The population doubles every 30 years. In the year 1990 the population of the island was 400 people.

- D. What will the population be in the year 2020? What will the population be in the year 2050?
- E. Estimate the year when there will be 6,000 people on the island. Show the work that leads to your estimate.
- F. Estimate the year when the needs of the population will exceed the available living space. Show your work and explain how you arrived at your estimate

RUBRIC

Notes:

- For a score of 2 or 3, an acceptable estimate in response to Part B must lie in the interval $2095 \leq \text{estimate} \leq 2110$; however, for a score of 4, 2110 is NOT acceptable. Acceptable estimates for Part C must lie in the interval $2125 \leq \text{estimate} \leq 2139$.
- For a score of 2 or 3 minor arithmetic or transcription errors may be present in the work for Part B or C.

Score Description

- 1 Correct numerical answers for both parts of Part A
 OR
 correct numerical answer for one part of Part A and the correct value given for island’s population capacity.
- 2 Correct numerical answers for both parts of Part A and an acceptable estimate for Part B or C with clear presentation of work
 OR
 acceptable estimates for Parts B and C with clear presentation of work.
- 3 Correct numerical answers for both parts of Part A and acceptable estimates for Parts B and C with presentation of work which may be only partially adequate
 OR
 except for the estimate for Part C, everything correct and supported by an adequate presentation of work.
- 4 Correct numerical answers to both parts of Part A and acceptable estimates for Parts B and C with clear presentation of work.

Note: See General Scoring Rubric for Written Response Items for further guidelines.

MDTP AR96ILND

Appendix 6

Sample Math Rubrics

De Anza College – Barbara Illowsky

Mathematics presentations rubric

Mathematics lab work rubric

Online mathematics rubric for discussion postings

Online mathematics rubric for discussion responses

Sierra College

Developmental Math Word Problem Rubric

Orange Coast College – Joan Cordova

Word problem sample to use with the rubric

The following is an example of a rubric Barbara Illowsky of De Anza College uses to grade take home projects and math labs including presentations as part of the work.

	Excellent	Good	Satisfactory	Unsatisfactory
Key concepts	The answers fully demonstrate that the student(s) understands the key concepts.	The answers mostly demonstrate that the student(s) understands the key concepts.	The answers somewhat demonstrate that the student(s) understands the key concepts.	The answers do not demonstrate that the student(s) understands the key concepts.
Mathematics language	The project includes mathematical terminology, notation, and labeling of units when appropriate.	The project mostly includes mathematical terminology, notation, and labeling of units when appropriate.	The project includes some mathematical terminology, notation, and labeling of units when appropriate.	The project misuses mathematical terminology or notation or does not label units when appropriate.
Strategies	The project shows complete evidence of appropriate strategies for solving the problem.	The project shows nearly complete evidence of appropriate strategies for solving the problem.	The project shows some evidence of appropriate strategies for solving the problem.	The project shows no evidence of using appropriate strategies for solving the problem.
Algorithms and computations	There are no significant factual errors and/or misconceptions in the algorithms or calculations.	There are only minor computational errors. There are no misconceptions in the algorithms.	There are some computational errors or misconceptions in the algorithms.	Most of the project shows computational errors and misconceptions in the algorithms.
Writing mechanics	Non-numerical answers are written in complete sentences, explaining what was done and why it was done.	Non-numerical answers are mostly written in complete sentences, explaining what was done and somewhat addressing why it was done.	Non-numerical answers are occasionally written in complete sentences. Explanations are vague.	Non-numerical answers are not written in complete sentences. Explanations are difficult to interpret.
College level presentation	Graphs are constructed accurately, including measuring and scaling, labeling of axes, straight lines (when applicable), and neatly. AND The project is neatly presented and organized. AND The project is turned in by the first 5 minutes of class on the due date. AND The papers are stapled (if more than 1 paper).	Graphs are accurately drawn but missing labeling. Labeling is included, but scaling is not accurate. Graphs are still neatly drawn. OR The project is fairly neatly presented and organized. OR The project is turned in by the end of class on the due date. OR The papers are NOT stapled (if more than 1 paper).	Graphs are missing many of the required parts or are not neat. OR The project is not neatly presented or not organized. OR The project is completed and turned in by the end of the day on the due date. OR The papers are NOT stapled (if more than 1 paper).	Graphs are drawn without straight edges (when applicable), are messy, are not accurate, or do not reflect the data or distribution. OR The project is neither neatly presented nor organized. OR The project is turned in after the due date. OR The papers are NOT stapled (if more than 1 paper).

The following is an example of a rubric Barbara Illowsky of De Anza College uses to grade take home projects of 3-5 days work.

Grading Rubric for Math Labs – 30 points

	Max	Excellent	Good	Satisfactory	Unsatisfactory	Your
	Points	Maximum Points			0 Points	Points
Key concepts	8	The answers fully demonstrate that the student(s) understands the key concepts.	The answers mostly demonstrate that the student(s) understands the key concepts.	The answers somewhat demonstrate that the student(s) understands the key concepts.	The answers do not demonstrate that the student(s) understands the key concepts.	
Detail and facts	8	The answers include full and adequate detail and have no significant factual errors and/or misconceptions.	The answers include some detail or have only minimal significant factual errors and/or misconceptions.	The answers include some detail and have only minimal significant factual errors and/or misconceptions.	The answers do not provide adequate detail and have several significant factual errors and/or misconceptions.	
Writing mechanics	8	Non-numerical answers are written in complete sentences, with proper grammar.	Non-numerical answers are written mostly in complete sentences, with proper grammar.	Non-numerical answers are occasionally written in complete sentences, with proper grammar in some places.	There are several incomplete sentences, cases of poor grammar.	
College level work	6	Graphs are constructed accurately, including measuring and scaling, labeling of axes, straight lines (when applicable), and neatly. AND The lab is neatly presented and organized. AND The lab is turned in by the first 5 minutes of class on the due date. AND The papers are stapled (if more than 1 paper).	Graphs are accurately drawn but missing labeling. Or, labeling is included, but scaling is not accurate. Graphs are still neatly drawn. OR The lab is fairly neatly presented and organized. OR The lab is turned in by the end of class on the due date. OR The papers are NOT stapled (if more than 1 paper).	Graphs are missing many of the required parts or are not neat. OR The lab is not neatly presented or not organized. OR The lab is completed and turned in by the end of the day on the due date. OR The papers are NOT stapled (if more than 1 paper).	Graphs are drawn without straight edges (when applicable), are messy, are not accurate, or do not reflect the data or distribution. OR The lab is neither neatly presented nor organized. OR The lab is turned in after the due date. OR The papers are NOT stapled (if more than 1 paper).	

These final rubrics from Barbara Illowsky of De Anza College is used to grade online discussions for an online math class.

Grading Rubric for Initial Discussion Posting – 5 points

	Excellent	Satisfactory	Unsatisfactory
Points →	1	0.5	0
Focus	The posting demonstrates that the student understands the key concepts.	The posting somewhat demonstrates that the student understands the key concepts.	The posting does not demonstrate that the student understands the key concepts.
Completeness	The posting includes examples when appropriate and has adequate detail.	The posting includes some examples and some detail.	The posting neither includes examples when appropriate nor does it provide adequate detail.
Detail and facts	The posting has no significant factual errors and/or misconceptions.	The posting has a few significant factual errors and/or misconceptions.	The posting includes many factual errors and/or misconceptions.
Mechanics	The posting is written in complete sentences and with proper grammar.	Most of the posting is written in complete sentences and with proper grammar.	There are several incomplete sentences and cases of poor grammar.
Deadline and length	The posting is completed on time and with a minimum of 100 words.	The posting is completed one to two days late or has fewer than 100 words.	The posting is more than 2 days late and/or has significantly fewer than 100 words.

Grading Rubric for Discussion Response – 5 points

	Excellent	Satisfactory	Unsatisfactory
Points →	1	0.5	0
Connection	The response concretely connects with the original posting.	The response somewhat connects with the original posting.	The response does not connect with the original posting.
Significance	The response adds significantly to the original posting.	The response adds somewhat to the original posting.	The response does not add to the original posting.
Contribution	The response contributes good suggestions to expand or improve the original posting.	The response contributes fair suggestions to expand or improve the original posting.	The response does not contribute good suggestions to expand or improve the original posting.
Mechanics	The response is written in complete sentences and with proper grammar.	Most of the response is written in complete sentences and with proper grammar.	There are several incomplete sentences and cases of poor grammar.
Deadline and length	The posting is completed on time and with a minimum of 50 words.	The posting is completed one to two days late or has fewer than 50 words.	The posting is more than 2 days late and/or has significantly fewer than 50 words.

Math Rubric Sierra College

(see next page for a sample word problems that can be used with this rubric)

DEVELOPMENTAL MATH WORD PROBLEM RUBRIC

Give one point to each blank (10 points)

1) VARIABLE STATEMENT

_____ Statement has enough detail to be interpreted easily.

_____ Statement is a statement and not a question.

_____ Statement is assigned a variable.

2) EQUATION

_____ Equation includes correct parentheses.

_____ Equation is written in correct order.

_____ Equation is assigned a variable.

3) COMPUTATION AND SOLUTION

_____ Math is done correctly to arrive at the correct answer.

_____ Answer is correct even though equation is wrong.

_____ Answer has correct units.

_____ Answer is assigned a variable.

EVALUATOR _____

Pre Score _____

Post Score _____

Date _____

Date _____

EVALUATOR _____

Pre Score _____

Post Score _____

Date _____

Date _____

Sierra College

General Math

(Rubric is put in instructor's syllabus for math work in general)

Student work in this course will be evaluated according to the following 5-point standard scale.

- 5 Excellent; Completely achieves all of the purposes of the task; Demonstrates full understanding without any deficiencies
- 4 Good; Adequately achieves all of the purposes of the task; Demonstrates understanding with some minor deficiencies
- 3 Satisfactory; Adequately achieves many of the purposes of the task; Demonstrates some understanding with some deficiencies
- 2 Unsatisfactory; Inadequately achieves the purposes of the task; Demonstrates partial understanding with fundamental deficiencies
- 1 Inadequate; Inadequately achieves the purposes of the task; Demonstrates little understanding with major deficiencies
- 0 Unacceptable; Purposes of the task are not accomplished; Unable to demonstrate understanding

Developmental Math Word Problem Rubric (Joan Cordova)

Sample Word Problem no calculator:

Two friends averaging 70 mph drove 1400 miles across the country. How many hours did they drive?

Solution:

$t =$ hours they drove	Statement detailed and easily understood, variable assigned
$d/r = t$	Equation Includes correct order, variable assigned
$1400/70 = t$	Computation Correct computation, correct units
$t = 20$ hours	Solution Correct units, variable assigned

Sample Word Problem allowing a calculator:

On average, a Toyota Prius gets 45 mpg and a Nissan Altima gets 33 mpg. If a driver drives 10,000 miles in a year and the cost of gas is \$3.50 how much money would be saved this year by driving the Prius? Round to the nearest cent.

Solution:

$S =$ the money saved	Statement detailed and easily understood, variable assigned
$(\text{Cost for Altima gas}) - (\text{Cost for Prius gas}) = S$	
$((10,000 * \$3.50) / 33) - ((10,000 * \$3.50) / 45) = S$	Equation Includes correct parentheses, correct order, variable assigned
$\$1060.61 - \$777.78 = \$282.83$	Computation Correct computation, correct units
$S = \$282.83$	Solution Correct units, variable assigned

Appendix 7
Sample Math Lessons with Active Learning
Lesson Plan Template

Title	Finding the Circumference of a Circle
Instruction Goals and Objectives	Develop the formula for the circumference of a circle using exploration, given only a piece of string.
Instructional Strategies	Use a string to measure the diameter. Using this measurement, find the number of diameter measurements it takes to get around the circle. Now find the ratio of the circumference to the diameter. Expand to the formula for the circumference of a circle.
Assessment	Complete a worksheet/handout on finding, a) circumference, given diameter b) diameter, given circumference.
Presentation	You will need, various lengths of string & at least 2 circular objects. then Allow students to explore & then present formula.

Lesson Plan Template

Title	Prime factorization
Instruction Goals and Objectives	To be able to write composite numbers in prime factored form
Instructional Strategies	Do a factor tree of a composite number, cover the primes with clear post-it notes writing the number on the top. the Take the post its and place them next to each other to display the prime factored form. $24 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{3}$
Assessment	we have 3 modes of assessment 1) observation of group work; ask each group member ^{for} a question 2) impressions of ^{the} presentations using the rubric on page 69 ch9 3) evaluate individual work for accuracy by quick visual inspection
Presentation	supplies needed: - small post-it notes - easel paper for each group - drill/practice worksheets

Group 1

Lesson Plan Template

Title	Place Value System
Instruction Goals and Objectives	To Show The Importance of Place Value
Instructional Strategies	<ol style="list-style-type: none">1. Distribute color coded 3x5 cards each containing a 4-digit number.2. Randomly select a color. Have student read the number.3. Assign the individual digits to a matching color column.4. The vertical addition will reinforce place value.
Assessment	Students will be asked to read their 4-digit numbers and to verify the place value of the digits.
Presentation	Use 3x5 cards in several colors and wall-mounted blank charts for recording responses.

Lesson Plan Template

Title	Number Line Rope
Instruction Goals and Objectives	Students will develop number sense, fraction sense, relative size of numbers, comparing fractions, estimation.
Instructional Strategies	students will be actively involved, and experience kinesthetic and contextual learning, use manipulatives, communicate verbally, collaborate, develop their math power, and participate in ESL strategies.
Assessment	In formally, students will understand the order of numbers, as well as magnitude, through visual comparison. A quiz or other more formal assessment may be given if desired, for evaluation.
Presentation	Use rope, index cards with numbers

Lesson Plan Template

Title	SYSTEMS OF EQUATIONS WITH COMBINED SPEEDS
Instruction Goals and Objectives	Solving Systems of equations involving $D = r \cdot t$
Instructional Strategies	<p>SLO: student, given distances and times will set up and solve a system of equations to find speed of boat and current (River)</p> <ul style="list-style-type: none"> • Interactive examples to investigate relationships between sp boat/current/combined speed • Build on that to set up system of linear equations to solve for unknown boat/current • Have students work on a) combined speed prob-
Assessment	<p>lems and b) system of equations</p> <p>last bullet is worksheet to fill in blanks for assessment</p>
Presentation	<ul style="list-style-type: none"> • Vugraph with objectives, SLO • charts for samples to fill in (what would be hand out)

Title: Graphing Linear Equations

Goal: Get students to understand what a graph is showing us in regard to a linear equation.

Objective: Have students discover or see for themselves that

- When you graph the solutions to a linear equation on a Cartesian graph, the points are all collinear.
- All points on the line implicitly mentioned above are solutions to the linear equation.

Instructional Strategies:

- Small-group activity – through handout
- Interactive lecturing
- Presentation
- Guided discovery
- Learning through questions

Assessment: Answers to questions 4 – 6 on activity handout.

Presentation:

- Remind and review two-variable equations and solutions.
- Review Cartesian graphs: plotting and reading coordinates.
- Organize groups and hand out activity.
- Students do activity.
- Group representatives present answers.
- Interactive lecture summary.

Prerequisite Skills:

- Able to tell when an ordered pair is a solution to a two-variable equation.
- Able to use substitution and equation-solving methods to find the missing half of an ordered pair solution to a linear equation in two variables.
- Able to read an x - y table and plot the points on a graph.
- Able to read a point on a graph and interpret it as a pair of x - y values.

Graphing an Equation

Find the solutions to complete this table for the equation $x - y = 3$.

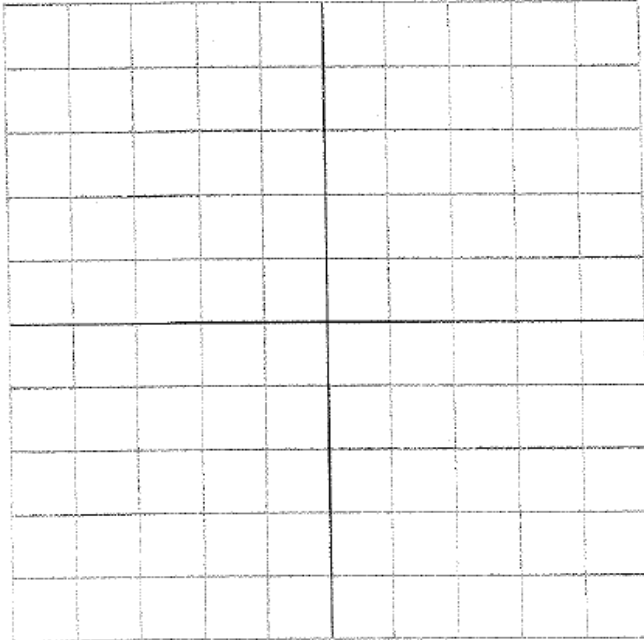
x	0		-1		1.5	
y		0		2		-0.5

Now answer these questions.

1. Your table above had six solutions to the equation $x - y = 3$. Does that equation have other solutions that are not on the table?
2. How many other solutions do you think that the equation $x - y = 3$ has?
3. What makes it difficult to show someone all the solutions to $x - y = 3$?

On The Graph

Label this graph properly.



Plot the six solutions for the equation $x - y = 3$ from your table on this graph.

Now answer these questions.

4. What kind of shape or pattern is being formed by the points you plotted?
5. Using the shape or pattern formed by the points, find another point that is part of the same pattern. Check: is it a solution to the equation $x - y = 3$?
6. If someone wants to know all the solutions to the equation $x - y = 3$, can you tell them? Can you show them?

Lesson Plan Template

Title	Graphing Linear Inequalities
Instruction Goals and Objectives	To understand the relationship between the graph and the solutions of a linear inequality in two variables.
Instructional Strategies	Remind students how to plot points, check solutions to inequalities, and graph lines. Group work, class discussion, some lecture, individual work.
Assessment	After the group activity part where all the points are placed on the axes, make sure the points are graphed & color-coded properly. Check the individual problems for completeness & correctness.
Presentation	Group activity: Students form groups of 2-3 members. Each group is given 3 points for each of two inequalities. For each inequality, each group will have a point that is a solution, one point on the line, and one point that is not a solution. These points are then graphed on axes (on the board) - solutions in one color, values that aren't solutions in another color, labeling points. After all points are labeled, address any mistakes, then ask for observations/conclusions. Discuss how the equation of the line is the boundary, the side with the solutions should be shaded, the form of the line equation, and whether the boundary line is dotted or solid.

Possibly go over another example, and then have the students try a problem on their own.

Lesson Plan Template

Title	Least Common Denominator Scavenger Hunt
Instruction Goals and Objectives	To teach students how to find the LCD given 2 fractions.
Instructional Strategies	<ol style="list-style-type: none">1. LCD Scavenger Hunt Prime factor 2 den. using factor trees. Using the prime factors, hunt for the denominator that has the most copies and use those primes in the LCD.2. Build the LCD with tiles to create a doorway large enough for both den. to fit.3. 32 day commitment to find LCD for 2 given fractions for each day.
Assessment	<ol style="list-style-type: none">1. Reinforce & assess with another "you try it" problem.2. Pair students to explain their processes used in the "you try it."
Presentation	<ol style="list-style-type: none">1. Flip charts & colored markers2. Post-its (optional for prime factors)3. Building tiles.4. 3x5 cards for "you try it" example

Appendix 8: Sample Contextualized Math Activities

Earth Day Quiz – Contextual Mathematics

Here's a math quiz developed by Kerin Keys and Anna Werner of City College of San Francisco
kkeys@ccsf.edu

Earth Day Quiz 2008

Directions: On the answer sheet at the bottom of the page, write the letter of the best answer for each question below. Tear off the answer sheet, complete the information requested, and turn it in at the Quiz Table, on Ram Plaza, on Earth Day - Tuesday April 22.

When you turn in your answer slip, you will be automatically entered in a raffle to WIN PRIZES!

1. The average American uses 159 gallons of water per day. The average person in half of the rest of the world uses 25 gallons per day. What percent more does the average American use?
a. 636% b. 5.36% c. 536% d. 6.36%
2. Fill in the blank with the correct symbol: Energy used by the U.S. _____ Energy used by all developing countries combined.
a. = b. > c. <
3. To produce each week's Sunday newspapers, approximately 500,000 trees must be cut down. Considering that a high density forest has 250 trees per acre, how many acres of forest is that per year?
a. 26,000,000 b. 104,000 c. 2,000 d. 24,000
4. The following environmentally related job uses math every day:
a. solar power engineer
b. environmental attorney
c. environmental policy analyst
d. all of the above
5. The EPA estimates that you can save 12% on your utility bills if you use energy efficient appliances and insulate your house or apartment. If an average household pays \$150 a month during half of the year (summer and winter months) and \$75 a month during the other half, how much savings is that in a year?
a. \$27 b. \$118 c. \$162 d. \$81

13. The circumference of the earth is approximately 24,900 miles at the equator. If we lay sheets of paper 11 inches long end to end, how many pieces of paper will it take to go around the earth?
 a. 131,472,000 b. 143,424,000 c. 1,577,664,000 d. 191,232,000
14. Based on your answer above, if Americans use a total of 4.3 billion sheets of paper per day, approximately how many times would that circle the earth (every day!)?
 a. 5 times b. 25 times c. 30 times d. 40 times
15. I can personally protect the environment as a math student by:
 a. using both sides of notebook paper and the clean side of used printer paper
 b. using a refillable lead pencil
 c. re-using folders from one semester to the next
 d. all of the above.

Answer Slip: Complete, tear off

4/22/08

1. ___ 2. ___ 3. ___ 4. ___ 5. ___ 6. ___ 7. ___ 8. ___ 9. ___ 10. ___ 11. ___ 12. ___ 13. ___ 14. ___ 15. ___

Name: _____ **Contact tel. or e-mail:** _____

Department: _____ **PRIZES will be awarded (must get 10 correct).**

Thanks for your quiz participation and for supporting Mother Earth

Sample Contextualized Math Problems

Names _____

Congratulations!

It is the year 2030 and you have just won the **Intergalactic Lotto Grand Prize**. You have a choice of two prizes.

Option A You will receive \$50,000 on the first day and for each day after, you will receive \$10,000 more than you received the day before for a total of 30 days. So, the 2nd day you receive \$60,000, the 3rd day \$70,000 and so on.

Option B You will receive 1 penny on the first day and for each day after, you will receive double the amount you received on the previous day for a total of 30 days. So, the 2nd day, you receive 2 cents; the 3rd day 4 cents; and so on.

- a. Let A_n represent the amount of money you receive on the n^{th} day under Option A and let B_n represent the amount of money you receive on the n^{th} day under Option B. Complete the following tables.

n = nth day	A _n (in dollars)		n = nth day	B _n (in cents)
1	50000		1	1
2			2	
3			3	
4			4	
5			5	
n			n	

- b. Write a formula for the amount of money you would receive on the n^{th} day for each option. Write each formula below.

A_n (in dollars) = _____

B_n (in cents) = _____

- c. Use your formula A_n to determine how much money you will receive on the 30th day under Option A. Show your work and write your answer in a sentence.
- d. Now determine the **total amount of money** you will receive under Option A for the 30 days. Your answer will be in dollars. Show your work and write your answer in a sentence.
- e. Determine the **total amount of money** you will receive under Option B for the 30 days. Your answer will be in cents. Convert this to dollars. Show your work and write your answer in a sentence.

Applications of Logarithms

Logarithmic Functions have applications, just as exponential functions do. Among the applications of logarithmic functions are Earthquake Intensities and Magnitude.

The intensity of an earthquake is a measure of the strength of an earthquake as measured by a seismograph. Earthquakes can be very small where we can hardly feel them, to very violent earthquakes which can kill thousands of people and cause widespread damage.

Earthquakes are measured using what is called its Richter Magnitude. The newspapers will always report the magnitude of an earthquake. The Richter Magnitude of an earthquake can be defined to be the common logarithm of the intensity of the earthquake.

$$M = \log I$$

1. What is the Richter Magnitude of an earthquake with an intensity of 10,000?
2. Suppose an earthquake was twice as strong as the earthquake above. In other words, its intensity was twice the intensity of the earthquake in #1. What would be the magnitude of this earthquake?
3. Suppose an earthquake was 10 times as strong as the earthquake in #1. What would be the magnitude of this earthquake?
4. The above formula can be used to find the magnitude if we know the intensity. It can also be used to find the intensity if we know the magnitude. The 1989 Loma Prieta earthquake had a Richter magnitude of 7.1. What was the intensity of this earthquake?
5. The earthquake in India 2001 had a Richter Magnitude of 7.9. What was the intensity of this earthquake?

6. How many times stronger was the 7.9 earthquake than the 7.1 earthquake? To get this, divide the intensity of the stronger earthquake by the weaker one.
7. Suppose an earthquake had a Richter Magnitude of 7.1. What would be the Richter Magnitude of an earthquake that was twice as strong?

Saving for retirement.

You are 32 years old and you have just gotten your first professional job. You have decided to put \$2000 into an IRA. Assume that the IRA will pay 9% interest compounded monthly.

1. Write an equation for the amount A of money you will have in your IRA after t years.
2. Use your equation to determine how much money this IRA will grow to by the time you retire when you are 62.
3. What is the doubling time for this investment? Solve an equation using ALGEBRA to obtain your answer.

4. You decide that you will put \$2000 **every year** into an IRA. For each IRA payment, determine how much this money will grow to by the time you are 62 and can retire. You will make your last payment at age 61. Complete the table below.

IRA #	Age when invested	Years until age 62	Amount at age 62
1	32	30	
2	33	29	
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			

5. How much money TOTAL will you have in all of your IRA accounts to live off of when you reach age 62?

Examples used by permission from Diane Mathios <mathiosdiane@deanza.edu>

Appendix 9 Reading and Mathematics

Seven Reading Problems

This is the handout that Sara Pries, Sierra College mathematics instructor uses to begin her workshop:

Directions: This is a “reading” test. Don’t let the numbers fool you. Think and picture rather than compute.

1. Does England have a fourth of July?
2. If you had only one match and entered a room where there was a lamp, an oil heater, and some kindling wood, which would you light first?
3. A woman gave a beggar 50 cents. The woman is the beggar’s sister, but the beggar is not the woman’s brother. Why?
4. Is it legal in North Carolina for a man to marry his widow’s sister?
5. A garden had exactly 50 different kinds of flowers, including 10 kinds of roses, 3 kinds of sweet peas, 2 kinds of alyssum, 5 kinds of carnations, 3 kinds of zinnias, 8 kinds of poppies, 4 kinds of snapdragons, 5 kinds of gladiolus, and 6 kinds of phlox. How many different kinds of flowers did the garden have?
6. A rooster is sitting on the peak of a roof and lays an egg. Which way does the egg roll: to the right or to the left?
7. Abbott, Baker and Casper are a detective, an entomologist, and a farmer, although not necessarily in that order. Abbott was the mother of healthy twins yesterday. Casper has a deadly fear of insects and will not even get close enough to kill if she sees it. The farmer is getting worried because she and her husband are getting old and will not be able to run the farm for too many more years, and she has no children. Casper, unmarried, especially likes to date brunettes. What is the occupation of each of the three women?

These are the handouts Lisa Rochford, Reading Instructor, gives out to discuss and model “**How to Annotate a Text**”, “**Word Problems Step-by-Step**”, and “**Five Word Problems: Practicing Pace, Annotate, Translate, and Paraphrase.**”

How to Annotate a Text

As an “active reader,” you already know that when you read textbook assignments, you should have questions in your mind. As you read, you should be looking for the answers to these questions. You

should also have a pencil in hand so that you can “annotate” your text. As the word suggests, you “take notes” in your textbook.

Unlike “highlighting,” which is a passive activity, the process of annotating text helps you to stay focused and involved with your textbook. You’ll find that the process of taking notes as you read will help you to concentrate better. It will also help you to monitor and improve your comprehension. Annotating will also help you find important concepts for review. If you come across something that you don’t understand or that you need to ask your instructor about, you’ll be able to quickly make note of it, and then go on with your reading.

The following is a list of some techniques that you can use to annotate text:

- Underline important terms
- Circle definitions and meanings
- Write key words and definitions in the margin
- Signal where important information can be found with key words or symbols in the margin
- Write short summaries in the margin at the end of sub-units
- Write the questions in the margin next to the selection where the answer is found
- Indicate the steps in a process by using numbers in the margin
- Develop a personal system of symbols and abbreviations in the margins to find information quickly:

Symbols you can use to annotate. There are, naturally, many more symbols that could work well, but these are a place to start:

* ? + - →

Abbreviations you can use to annotate:

Def-definition

Sum-summary or summary statement

Ex-example

Questions to ask yourself:

Have you noted the source and date of the material or lecture?

Have you accurately captured all of the main ideas?

Can you summarize or paraphrase the material?

Word Problems Step-by-Step

The most important issue with any difficult reading problem is to establish a meaningful process. Here are some steps to think about in establishing your own process:

Before you begin, make sure you do two things:

A. You THOROUGHLY review the math concepts you will be learning

B. You know the terminology of the discipline

1. Pace (AKA, Reading Speed)

Make sure you slow down enough to read every word. In any chunk of text, there are more important items and less important items. An important rule of thumb is “The more difficult the text, the slower your pace.”

TIP: Touch every word with the tip of your pen or pencil. This will make sure your eye catches every word and will help you slow down.

TIP: You will need to read the problem at least twice. Read it slowly, using the above tip, before you do anything else. That way, you will get the actual comprehension out of the way.

II. Annotate (AKS, Mark the page)

If you annotate as you go, you are processing the information twice. Once to understand the words themselves and again to judge whether you should mark it or not. Consider crossing out unnecessary or introductory information.

TIP: Use a highlighter, different colored pen, or a system of notations (circles, stars, etc.) to draw your eye to the important pieces of information in the problem.
ALWAYS DO THIS ON YOUR SECOND READ THROUGH.

Make sure you read the problem through at least once before you begin to annotate

III. Translate (AKA, Say It In Math Language)

Take your annotations and carefully translate them into math language.

TIP: Make sure you have a thorough understanding of the math concepts you are learning before you begin your word problems. A review of terms (sum, add, subtract, divide) before you attempt a word problem is a very good idea.

SEE STEPS TO COMPLETE BEFORE YOU START

IV. Paraphrase (AKA, Say IT in Your Own Words)

Before you begin to solve your equation, see if you can say it to yourself using different words. If you can do this easily, you understand what you are doing. If you can't, you need to review. This step can also include visualization, drawing pictures or any other method you know that will allow you some ownership of the information you need.

TIP: Again, using your pencil, make sure you have accounted for everything you marked in your equation. Make necessary adjustments.

Five Word Problems: Practicing Pace, Annotate, Translate, and Paraphrase

1. Jim bought an 11-piece set of golf clubs for \$120, one dozen golf balls for \$9, and a pair of golf shoes for \$45. How much did he spend in all?
2. A hospital had 20 bottles of thyroid medication with each bottle containing 2,500 5-gram tablets. It gave 5 bottles of tablets to the Red Cross. How many tablets does the hospital have left?
3. A roofing company has purchased 1,134 squares of roofing material. One square measures 10 feet by 10 feet. If each cabin needs 9 squares of material, find the number of cabins that can be roofed.
4. A theatre owner wants to provide enough seating for 1,250 people. The main floor has 30 rows of 25 seats in each row. If the balcony has 25 rows, how many seats must be in each balcony row to satisfy the owner's requirements?
5. A Boeing 747 traveling 675 miles per hour carried 254 passengers. After three hours, it landed in Atlanta where 133 passengers deplaned before it continued on to Washington, D.C., its final destination, 900 miles away. How many passengers deplaned in Washington?

Math Facts Information

Lynn Hargrove, Mathematics Instructor, gives out and discusses key words and other information with the following handouts.

Problem Solving

There are five general rules to follow when trying to solve word problems. They are:

1. Read through the entire problem from beginning to end at least once. Some people miss important facts because they "skim" the problem rather than actually reading it.
2. Look for the main idea. In word problems, it is usually the question. At this step, you are looking for what it is that the question wants you to find.
3. Decide on the operation. There could also be a combination of operations you need to use to answer the question. Look for key words. Draw pictures if necessary to help you "see" the problem.

Some examples of key words are

- **Addition:** in all, add, altogether, total, sum, combine
- **Subtraction:** difference, minus, more than, less than, subtract, take away
- **Multiplication:** product, times, of
- **Division:** how many are in, how many times smaller or larger, average, quotient

4. Pick out the details that will support your operation. Be careful. Many times extra information is added that has no bearing on solving the problem.

5. Solve the problem and label the answer.

Problem-Solving Words

Math Symbols

Key Words

+	The sum of, plus, added to, joined with, increased by, more than, more
-	The differences of, minus, subtracted from, take away, decreased by, less than, less, reduced by, diminished by, exceeds
X	The products of, times, multiplied by, equal amounts of, goes into, over
=	The same as, is equal to, equals, is, was, are, makes, gives the result of, leaves, will be
2●	Twice, double, two times, twice as much as
$\frac{1}{2}$ ●	Half, one-half times, half as much as n (any letter). What number, what part, a number, the number, what amount, what percent, what price
$2 \bullet n$	Twice a (the) number, double a (the) number
$\frac{1}{2} \bullet n$	Half a (the) number, one-half a (the) number
\div	The quotient of, how many are in, how many times smaller or larger, average

Tips for Solving Word Problems Involving Multiplication and Division of Fractions

General Guidelines:

1. Read the problem and ask yourself what you are looking for.
2. Always look for indicator words. However, you may not always find these indicator words.
3. Use at least three steps. To set up the problem, work it and check it.
4. If necessary change the fractions to whole numbers and work that way. Then go back and use the fractions.
5. Does your solution make sense to you?

Multiplication Tips:

1. Remember that multiplication is repeated addition.

Example: For an annual pancake breakfast $\frac{2}{3}$ cup of Bisquick is needed per

person to make the pancakes. If about 135 people are expected to attend, then how much Bisquick will you need?

1a) x = total amount of Bisquick needed for pancake breakfast

$$1b) x = \frac{2}{3} \cdot 135$$

$$1c) x = 90 \text{ cups}$$

2. If the word “of” is used most times it means multiplication. Usually you are finding a fraction of something which means to multiply. The word “of” must follow a fraction.

Example 1: Sociologists have discovered that $\frac{2}{5}$ of the people in the world are shy. A

sales manager is interviewing 650 people. How many of these people might be shy?

(Notice the word “of” follows a fraction)

X = the number of shy people being interviewed

$$X = \frac{2}{5} \cdot 650$$

$$X = 260 \text{ people}$$

Example 2: After Jack takes the CBEST teaching exam he can earn \$88 working a full day as a substitute teacher. How much would he receive if he works $\frac{3}{4}$ of a day?

A = amount earned working

$$A = \frac{3}{4} \cdot 88$$

$$A = \$66$$

3. Most times when you have information given that has *different measures*, you will multiply.

- a. Example 1: How much salmon is needed to serve 30 people if each person gets $\frac{2}{5}$ pound? (Notice you have servings and pounds and you need to find pounds for your answer).

X = pounds of salmon needed

$$X = \frac{2}{5} \cdot 30$$

$$X = 12 \text{ lb.}$$

- b. A sandwich shop sells submarine sandwiches by the foot. If one serving is $\frac{2}{3}$ foot long, how many feet would you need to feed 30 people? (Notice the measures are different)

N = number of feet of sub sandwich needed

$$N = 30 \cdot \frac{2}{3}$$

$$N = 20 \text{ ft.}$$

4. The measures are *always the same* when finding area of a figure.

Example: Find the area of the backyard if it measures $14\frac{2}{3}$ feet by $10\frac{2}{5}$ feet.

A = area of the backyard

$$A = 14\frac{2}{3} \cdot 10\frac{2}{5}$$

$$A = 152\frac{8}{15} \text{ sq. ft.}$$

Division of Fractions Tips:

1. Be careful because order does make a difference and is important!
2. If the words say split, divide, taken off, break into groups or anything hinting at taking apart, then you divide.

3. a. Example 1: The school district purchased $\frac{3}{4}$ ton of clay. The clay is to be distributed equally among the district's 6 schools. How much does each school receive? (Key word: distributed equally means divide)

X = amount of clay each school receives

$$X = \frac{3}{4} \div 6$$

$$X = \frac{1}{8} \text{ ton of clay}$$

4. When it seems that you “take away” repeated amounts from the whole, you divide. Also notice that the measures are the same and the answer is a different measure.

Example 1: The air guard uses \$9 million to spend on new helicopters. If each helicopter costs $\frac{3}{4}$ million, how many helicopters can be bought?

(Notice repeated “take aways”. You start with \$9 million and take away how much till there is none left).

N = number of helicopters to be bought

$$N = 9 \div \frac{3}{4}$$

$$N = 12 \text{ helicopters}$$

Example 2: The market prepackages Swiss cheese into $\frac{3}{4}$ pound packages.

How many packages can be made from a 15 pound block of Swiss cheese?

X = number of packages of cheese

$$X = 15 \div \frac{3}{4}$$

$X = 20$ packages

Solving Application Problems

Solving with One Unknown

I. Translating into a Variable Statement

A. Addition

<u>Phrase</u>	<u>Algebraic Expression</u>
1. the sum of a and 8	$a + 8$
2. 4 plus c	$4 + c$
3. 16 added to m	$m + 16$
4. 4 more than n	$n + 4$
5. 20 greater than m	$m + 20$
6. t increased by r	$t + r$
7. exceeds y by 35	$y + 35$

B. Subtraction

<u>Phrase</u>	<u>Algebraic Expression</u>
1. the difference of 23 and p	$23 - p$
2. 550 minus h	$550 - h$
3. w less than 108	$108 - w$
4. 7 decreased by j	$7 - j$
5. m reduced by x	$m - x$
6. 12 subtracted from g	$g - 12$
7. 5 less f	$5 - f$

C. Multiplication

<u>Phrase</u>	<u>Algebraic Expression</u>
1. the product of 4 and x	$4x$
2. 20 times b	$20b$

- | | |
|-----------------------|----------------|
| 3. twice r | $2r$ |
| 4. $\frac{3}{4}$ of m | $\frac{3}{4}m$ |

D. Division

<u>Phrase</u>	<u>Algebraic Expression</u>
1. The quotient of r and 10	$r \div 10$ or $\frac{r}{10}$
2. a divided by b	$a \div b$
3. the ratio of c to d	$c \div d$
4. k split into 4 equal parts	$k \div 4$

II. Combining variable expressions with more than one operation.

<u>Phrase</u>	<u>Algebraic expression</u>
1. Sum of twice a number and 9	$2n + 9$
2. Opposite of a number decreased by 5	$-n - 5$
3. Sum of opposite a number and -5	$-n + -5$ or $-n - 5$
4. Product of twice a number and 8	$(2n)(8)$
5. Five times the sum of twice a number and -5	$5(2n - 5)$
6. Six times the sum of twice the opposite of a number and -8	$6(-2n - 8)$
7. Product of 7 and the sum of a number and ten	$7(n + 10)$
8. Sum of 3 times a number and -4 multiplied by 5	$3n - 4(5)$
9. Sum of -10 and 6 times the opposite of a number	$-10 - 6n$
10. The product of 7 and 6 less than a number	$7(n - 6)$

II. Algebraic Equations

Procedure

1. Choose a variable to represent what is missing in the problem.
2. Write an equation using the variable.
3. Solve the equation.

Examples

1. Four more than 6 times a number is the same as 9 times the number increased by 10.

Find the number.

a) $n =$ the missing number

b) $4 + 6n = 9n + 10$

c) $n = -2$

2. A number plus 5 more than 3 times the number is 27.

a) $n =$ a number

b) $n + 5 + 3n = 27$

c) $n = 5.5$

Solving Applications Problems with Two Unknown Quantities

I. Hidden Values: Coin Problems

- A. Remember that when using coins that each has a decimal value when thinking in terms of dollars.

a. Pennies = \$0.01

b. Nickels = \$0.05

c. Dimes = \$0.10

d. Quarters = \$0.25

e. Half dollars = \$0.50

- B. When talking about how many of each coin you have and you have a total value in dollars, don't forget to put the decimal amount next to the coins. For example:

If you have a total of \$4.25 in quarters and dimes, remember:

$$(\$0.25 \times \text{quarters}) + (\$0.10 \times \text{dimes}) = \$4.25$$

$$0.25q + 0.10d = \$2.45 = \$4.25$$

- C. **Coin Problem:** Elaine has quarters and dimes totaling \$2.55. If she has one more dime than quarters, how many of each does she have?

	Amount	x	Value	=	Total
quarters	q		0.25		$0.25q$
dimes	$(1 + q)$		0.10		$0.10(1 + q)$
mix					2.55

a. $q =$ quarters

$d =$ dimes $= 1 + q$

b. $0.10d + 0.25q = 2.55$

$$0.10(1 + q) + 0.25q = 2.55$$

$$0.10 + 0.10q + 0.25q = 2.55$$

$$0.10 + (0.10q + 0.25q) = 2.55$$

$$0.10 + 0.35q = 2.55$$

$$\underline{- 0.10} \qquad = 0.10$$

$$\underline{0.35q = 2.45}$$

$$\frac{0.35}{0.35} \quad \frac{2.45}{0.35}$$

$$q = 7$$

quarters

$$\begin{aligned} c. q &= 7 \text{ quarters} \\ d &= 1 + q \\ &= 1 + 7 \\ &= 8 \text{ dimes} \end{aligned}$$

II. Geometry Problem

A. Procedure:

1. Use the text to look up formulas for perimeter, area, volume, and circumference for various geometric figures.
2. Use substitution to solve the given geometric problems.
3. Make sure to label answers with the correct label.
 - a. Use the measure for perimeter and circumference.
 - b. Use square units for area.
 - c. Use cubic units for volume.

B. The width of a rectangle is 3 feet less than the length. If the perimeter is 22 feet, what are the length and the width of the rectangle? (sketch the figure)

1. What information is given and what is it you are looking for?

- a. Given: $W = \text{width} = L - 3$

$$L = \text{length}$$

$$P = \text{perimeter} = 22 \text{ ft.}$$

$$L - 3$$

L



- b. $P = 2L + 2W$

2. Three Steps

- a. $L = \text{Length}$

$$L - 3 = \text{Width}$$

- b. Equation: $22 = 2L + 2(L - 3)$

Work:

$$22 = 2L + 2(L - 3) \qquad W = L - 3$$

$$22 = 2L + 2L - 6 \qquad 7 - 3$$

$$22 = 4L - 6 \qquad 4$$

$$\underline{+ 6 = + 6}$$

$$\underline{28 = 4L}$$

$$4 \quad 4$$

$$7 = L$$

- c. State the solution: $L = 7 \text{ feet}$ $W = 4 \text{ ft.}$

III. Work-Rate Problems

A. Procedure

1. Formulas: **Rate = $\frac{\text{work done in one situation}}{\text{Time it takes in one situation}}$**

$$\text{Work completed} = (\text{rate of work}) \times (\text{time})$$

2. First calculate the rate for each person. Then, use the work completed formula to figure out the missing factors. Use a chart if possible to figure out information.
- B. It takes an experienced carpenter 3 days to build a wooden deck on the back of a house. It takes an apprentice 4 days to do the same job. How long would it take for them to do the job together?

1. Given information: Experienced carpenter takes 3 days.

Apprentice carpenter takes 4 days.

T = time to do the job together

1 = indicates one complete job

	Rate	x	Time	=	Work
<u>completed</u>					
Experienced Carpenter ($\frac{1}{3}$ of the job in 1 day)		T			$\frac{1}{3}T$
Apprentice Carpenter ($\frac{1}{4}$ of the job in 1 day)		T			$\frac{1}{4}T$

2. Variable statement: T = time to complete the job together

3. Equation:
$$\frac{1}{3}T + \frac{1}{4}T = 1$$

4. Solve:
$$\frac{1}{3}T + \frac{1}{4}T = 1$$

$$\frac{4}{4}\left(\frac{1}{3}\right)T + \frac{3}{3}\left(\frac{1}{4}\right)T = 1$$

$$\frac{4}{12}T + \frac{3}{12}T = 1$$

$$\frac{7}{12}T = 1$$

$$\left(\frac{12}{7}\right)\left(\frac{7}{12}\right)T = (1)\left(\frac{12}{7}\right) \qquad T = \frac{12}{7}$$

5. Solution: T = 1 5/7 days

Mixtures

- A. Procedure: Two or more components are combined to produce a mixture with a certain value.
1. Read information and make a table to determine the amount of each component and the value of each component.
 2. There might be hidden components with cost and amount bought, so be careful.
- B. Coffee Grounds Inc. has two kinds of coffee. Coffee A costs \$9 per kg. and Coffee B costs \$6 per kg. If you buy 100 kg. of Coffee A, how many kilograms of Coffee B should be combined to obtain a blend worth \$1200?

Coffee	Amount	Value	Total
A	100	9	100(9)
B	b	6	6b
Mix		1200	9(100) + 6b

1. Variable statement: b = amount of Coffee B

2. Equation $9(100) + 6b = 1200$

$$\begin{array}{r}
 900 + 6b = 1200 \\
 - 900 \quad = - 900 \\
 \hline
 6b = 300 \\
 \frac{6b}{6} = \frac{300}{6} \\
 b = 50
 \end{array}$$

3. Solution: b = 50 kg.

C. The manager of a movie theater noted that 411 people attended a movie but neglected to note the number of adults and children. Admission was \$7.00 for adults and \$3.75 for children. The receipts were \$2678.75. How many adults and children attended the movie?

Movie Attendees	Amount	Value	Total
Adults	a	\$7.00	7 a
Children	c	\$3.75	3.75c
Mix	a + c = 411	\$2678.75	7a + 3.75c = 2678.75

1. Variable statement: a = adults

c = children

2. Equations: a + c = 411

$$\begin{array}{r}
 -c = -c \\
 \hline
 a = 411 - c
 \end{array}$$

$7a + 3.75c = 2678.75$

$$\begin{array}{r}
 7(411 - c) + 3.75c = 2678.75 \\
 2877 - 7c + 3.75c = 2678.75 \\
 2877 - 3.25c = 2678.75 \\
 - 2877 \quad = -2877 \\
 \hline
 -3.25c = -198.25 \\
 \frac{-3.25c}{-3.25} = \frac{-198.25}{-3.25}
 \end{array}$$

$$c = 61$$

$$a = 411 - 61$$

$$a = 350$$

3. Solution: $a = 350$ adults
 $c = 61$ children

V. Distance Problems: $D = R \times T$

A. The distance between Houston, Texas and Austin, Texas is 180 miles. A car

leaves Houston traveling toward Austin at an average rate of 68 miles per hour. At the same time, a van leaves Austin traveling toward Houston at an average rate of 52 miles per hour. Assuming they are traveling on the same route, how long will it take until they meet?

/

d r t

Car			
Van			

B. Keith leaves his home in Sacramento traveling east on I-80 at an average rate of 65 miles per hour. Three hours later his wife leaves home and takes the same route traveling at an average rate of 70 miles per hour. How many hours will it take his wife to catch up to him?

d r t

C. Bret drove for 4 hours on the freeway, then decreased his speed by 20 MPH and drove for 5 more hours on a country road. If his total trip was 485 miles, then what was his speed on the freeway?

d r t

Three=Step Problem Solving Procedure

First: Variable Statement

Second: Equation

Third: Solution stated with labels.

Example: You want to buy several items that cost \$245, \$123, and \$678. What is the total cost of the items?

- a. $c = \text{total cost}$
- b. $c = 245 + 123 + 678$

Work:

$$\begin{array}{r} 245 \\ 123 \\ + 678 \\ \hline 1046 \end{array}$$

c. $c = \$1046$

Solving Algebraic Equations

- Whatever is done to one side of the equation it must be done to the other side of the equation, too.**
- Addition and subtraction problems. (Addition Property of Equality)

a.
$$\begin{array}{r} 5 + a = 12 \\ -5 \quad = -5 \\ \hline a = 7 \end{array}$$

b.
$$\begin{array}{r} y - 10 = 8 \\ + 10 = +10 \\ \hline y = 18 \end{array}$$

- Multiplication problems. (Multiplication Property of Equality)

a.
$$\begin{array}{r} 4x = 20 \\ \hline 4 \quad 4 \\ \hline x = 5 \end{array}$$

b.
$$\begin{array}{r} 120 = 5y \\ \hline 5 \quad 5 \\ \hline 24 = y \end{array}$$

Three-Step Problem Solving Procedure for Proportions

First: Variable Statement

Second: Equation

Third: Solution stated with labels.

Example: It was time for your notebook check and you completed everything but your test corrections. This means that you have lost 5 points. What was your percentage score if the total points you could earn was 25 points?

- a. $p =$ your percentage score (%)
- b. $\frac{100 \text{ perfect \%}}{25 \text{ perfect points}} = \frac{p \text{ your \%}}{(25-5) \text{ your points}}$

Work: $25p = 100(25-5)$

$$25p = 100(20)$$

$$25p = 2000$$

$$\underline{25p = 2000}$$

$$25 \quad 25$$

$$p = 80\%$$

- c. $p = 80\%$

Appendix 10

Resources for Chapter 9

- Angelo, T. A., and Cross, K. P. (1993). *Classroom assessment techniques: A handbook for college teachers*. San Francisco: Jossey-Bass. Retrieved June 30, 2008, from <http://honolulu.hawaii.edu/intranet/committees/FacDevCom/guidebk/teachtip/assess-2.htm>.
- Bachford, J & Slater, D. (2008) *Assessing and improving student outcomes: What we are learning at Miami Dade college*. Community College Research Center (CCRC): Columbia University Teaching College Retrieved June 30, 2008, from http://www.achievingthedream.org/publications/research/Bashford_paper.pdf
- Bransford, J.D. (et al.) editors. (1999). *How people learn: Brain, mind, experience, and school*. Committee on Developments in the Science of Learning and Committee on Learning Research and Educational Practice, Commission on Behavioral and Social Sciences and Education, Washington, D.C. National Research Council. Available at <http://www.nap.edu/html/howpeople1/>.
- Burns S.M., Griffin, P., & Snow, C.E. (Eds). (1999). *Starting out right: A guide to promoting children's reading success*. Washington DC: National Academy Press.
- California Community College Chancellor's Office (CCCCO). (2008). MIS Datamart information. This data is available by doing a data mart search in the data base at <http://www.cccco.edu/SystemOffice/Divisions/TechResearchInfo/MIS/DataMartandReports/tabid/282/Default.aspx>.
- Center for Public Education. (2005). *Key lessons: Class size and student achievement*. Retrieved from http://www.centerforpubliceducation.org/site/c.kjJXJ5MPIwE/b.1533775/k.84C7/Key_lessons_Class_size_and_student_achievement.htm.
- Graves, S. L. (1998) *Success in Postsecondary Developmental Mathematics: A Curriculum Evaluation*. Institution Center for Occupational Research and Development, Inc. ED 460 691JC 980 382 retrievable at http://www.eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/19/b1/a0.pdf.
- Handelsman, J., Miller, S., and Pfund, C. (2008). *Scientific teaching*. The Wisconsin Program for Scientific Teaching. W.H.Freeman and Co.
- Hopper, C. (2007). *Math study skills: Diagnostic inventory*. Practicing College Learning Strategies, 4th edition, Houghton Mifflin.
- Intersegmental Committee of Academic Senates (ICAS). (2003). *Statement on competencies in mathematics expected of entering college students*. Retrieved June 30, 2008, from <http://www.asccc.org/icas/Resources.html>

Meehan, K. & Huntsman, H. (2004) *Pathways through algebra project*. Retrieved June 30, 2008, from http://www.ijournal.us/issue_09/ij_issue09_09_MeehanAndHuntsman_01.html.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

O'Connor, J. & Robertson, E. (2002). *MacTutor: History of mathematics – George Pólya* Retrieved July 16, 2008, from <http://www-gap.dcs.st-and.ac.uk/~history/Printonly/Polya.html>

Pólya, G. (1957). *How to solve it*. Garden City, NY: Doubleday and Co., Inc (A summary can be found at Alfred, P. website Understanding Mathematics retrieved June 6, 2008, at <http://www.math.utah.edu/~pa/math/polya.html>)

The Trustees of the California State University and the Regents of the University of California. (2008). *Mathematics diagnostic testing project (MDTP)*. Retrieved December 11, 2008, from <http://mdtp.ucsd.edu/>

The Conference Board, Inc (2006). *Are they really ready to work? : Employers' perspectives on the basic knowledge and applied skills of new entrants to the 21st century U.S. workforce*. The Workforce Readiness Project. Retrieved November 20, 2008, from http://www.conference-board.org/pdf_free/BED-06-Workforce.pdf.

U.S. Department of Education. (2008). *The final report of the national mathematics advisory panel*. Retrieved June 30, 2008, from <http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf> .

Website Resources

- Amatyc found at <http://www.amatyc.org/>
- Developmental Math website for Amatyc <http://www.devmath.amatyc.org/>
- MAPS at West Valley Mission contact is Linda Retterath - linda_retterath@wvm.edu <http://www.missioncollege.org/Depts/Math/MAPS/index.html>.
- <http://www.ltconline.net/greenl/SLO/SLOs/Math/MATSLOsStatewide.htm>
- Pólya autobiography at <http://www-gap.dcs.st-and.ac.uk/~history/Printonly/Polya.html>
- CMC³ Websites [Www.cmc3.org](http://www.cmc3.org) including the southern group [Www.cmc3s.org](http://www.cmc3s.org)
- [Www.nctm.org](http://www.nctm.org)