To sketch and analyze graphs of polynomial functions the following **Avenues** can be approached:

- A **polynomial function** is continuous and has no sharp edges. The function can be **increasing, decreasing or constant** and can have a **relative maximum and minimum**

The simplest graphs are power functions.

1. If the **degree of the function is eve**n the graph **touches** the x-axis
2. If the **degree of the function is odd** the graph **crosses** the x-axis

- The **leading coefficient test** allows to predict how the graph will rise or fall without bound
  1. When the degree is **odd and the leading coefficient positive** the graph falls to left and rises to the right
  2. When degree is **odd and the leading coefficient negative**, the graph rises to left and falls to right
  3. When degree is **even and the leading coefficient positive** the graph rises both sides
  4. When degree is **even and the leading coefficient negative** the graph falls both sides

- The **Fundamental Theorem of Algebra** states that at least on zero of the complex number system is a zero of the **nth** degree polynomial function.

- If x = a is **a zero** x = a is also **a solution of the function** f(x) = 0, **x –a is a factor**, (**a, 0) is an x-intercept of the graph f(x)**

- A polynomial of **n** degree has **n -1 turning points**

- A factor $(n-1)^k$ **k< 1** yields a repeated zero of multiplicity k. If **k is odd** the graph **crosses** the x-axis, if it is **even** the graph **touches** the x-axis

- **Intermediate Value Theorem**

If a and b are real numbers in an interval $[a,b]$ there need to be a value of the function between these values.

- **Linear Factorization Theorem**
- If a polynomial has **n** degrees will have also **n linear factors**
- Every polynomial can be written as the **product of linear and quadratic factors** with real coefficients, where the quadratic factors have no real zeros.

- The **rational zero test** $\frac{p}{q} = \frac{\text{factors of the constant}}{\text{factors of the leading coefficient}}$

allows to list all possible rational zeros

- Doing **consecutive long term or synthetic division** using the possible rational zeros as divisor can be established which are zeros for the given polynomial function. Using the quadratic formula can be found the irrational zeros or complex zeros.

  Complex zeros occur in **conjugate pairs.**

- **Descartes's rule of sign states:**
- The number of **positive zeros** is equal or less than that number by an even integer with the **number of changes in sign** of the polynomial
- The number of **negative zeros f(-x)** is equal or less than that number by an even integer with the **number of changes in sign** of the polynomial

- **Upper and Lower bound rule states:**
  - If f(x) is divided by x – c using synthetic division
  - **c > 0** and last row of the synthetic division is **positive c** is an **upper bound**
  - c > 0 and last row of the synthetic division is **alternately positive and negative c** is the **lower bound**

  - **Using tables of values for the domain and range can also help to plot the points and connecting them to sketch the graph**