Techniques for writing partial fractions decomposition

A **rational expression** can be written as a sum of two or more simpler rational expressions. This procedure is called **partial fraction decomposition**.

$N(x)$ = the polynomial in the numerator

D(x) = the polynomialin the denominator

1. **Distinct linear factors**

When the fraction degree is smaller than the degree of the denominator, factor out the denominator in linear factors and assign a constant for each numerator

$$\frac{x+7}{x^2-x-6}=\frac{A}{x-3}+\frac{B}{x+2}$$

Finding the LCD and cleaning the fractions we have:

$$x+7=A(x+2)+B(x-3)$$

To determine the values of the constants make A =0 by substituting

x =-2  and then substitute x = 3 to make B = 0

Therefore A= 2 and B = -1

To check the answer substitute the values of A and B so the decomposition will be

$$\frac{x+7}{x^2-x-6}=\frac{2}{x-3}+\frac{-1}{x+2}$$

Combining the right side results:

$$2(x+2)+(-1)(x-3)=x+7$$

Therefore the right side equals the left side of the equation.

2. **Repeated Linear Factors:**

When the degree of the numerator is greater than degree of the denominator

Divide the denominator into the numerator

$$\frac{x^4+2x^3+6x^2+20x+6}{x^3+2x^2+x}$$

$$x^3+2x^2+\frac{x\overline{\big)x^4+2x^3+6x^2+20x+6}^{\,x}}{5x^2+20x+6}$$

Rewriting the ratio between the quotient and the original denominator we have:

$$\frac{5x^2+20x+6}{x^3+2x^2+x}$$

Factor the denominator:

$$x(x^2+2x+1) \text{ or } x(x+1)^2$$

Include a constant for each power of x and (x+1) or

$$\frac{5x^2+20x+6}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

Finding the least common denominator, multiplying the numerators by the LCD and cleaning the fractions

$$5x^2+20x+6=A(x+1)^2+B(x+1)+Cx$$

If x = -1 will eliminates A and B and ***C = 9***
If x = 0 eliminates B and C and **A = 6**
Using any value for x and using A = 6 and C = 9 B = -1
Checking the answer:

We find that the sides of the equation are equal:

$$\frac{x^4+2x^3+6x^2+20x+6}{x^3+2x^2+x}=x+\frac{6}{x}+\frac{-1}{x+1}+\frac{9}{(x+1)^2}$$

Distinct Linear and Quadratic Factors

If the denominator can be factored in a linear and a quadratic equation we will include one partial fraction with a constant numerator and one with a linear numerator.

$$\frac{3x^2+4x+4}{x^3+4x} \quad \text{factoring out the denominator} \quad \frac{3x^2+4x+4}{x(x^2+4)}=\frac{A}{x}+\frac{Bx+C}{x^2+4}$$

Finding the LCD $3x^2+4x+4=A(x^2+4)+(Bx+C)x$

Collecting like terms : $3x^2+4x+4=Ax^2+4A+Bx^2+Cx$

$(A+B)x^2+Cx+4A$

Two polynomials are equal if the coefficient of like terms are equal

$3x^2=(A+B)x^2$ therefore $3=A+B$

: 4=C

4=4A

Therefore A=1 C=4 B=2

$$\frac{3x^2+4x+4}{x^3+4x}=\frac{1}{x}+\frac{2x+4}{x^2+4}$$

Repeated Quadratic Factors

When in the denominator we have a power raised to a power we include on partial fraction with a linear numerator for each power of the expression in the paranthesis

$$\frac{8x^3+13x}{(x^2+2)^2}$$

Writing the partial fraction decomposition $\frac{8x^3+13x}{(x^2+2)^2}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{(x^2+2)^2}$

Multiplying by the LCD and collecting like term we get:

$$8x^3+0x^2+13x+0=Ax^3+Bx^2+(2A+C)x+(2B+D)$$

Equating coefficients of like terms we get:

A=8, B=0, 2A+C=13, 2B+D=0

Therefore C=-3,D=0, A=8, B=0

For proof we write: $\frac{8x^3+13x}{(x^2+2)^2}=\frac{8x}{x^2+2}+\frac{-3x}{(x^2+2)^2}$ Therefore the right side equal with the left side of the equation